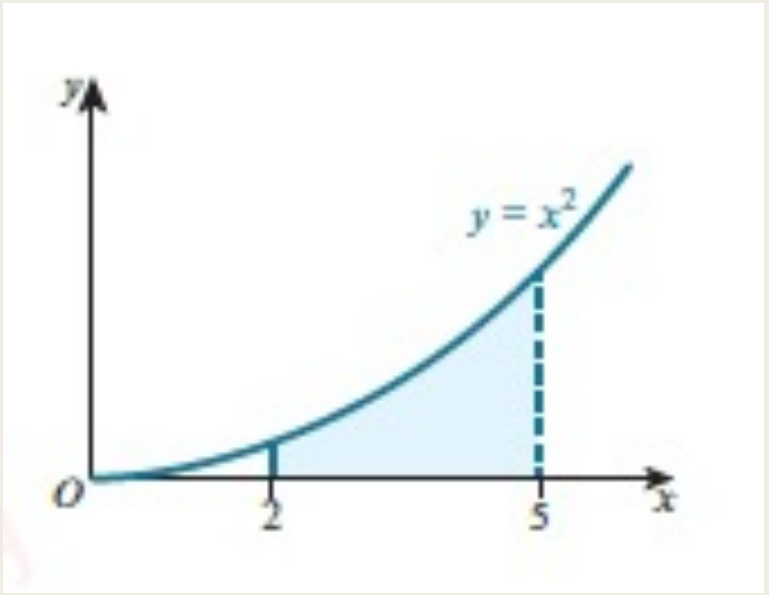
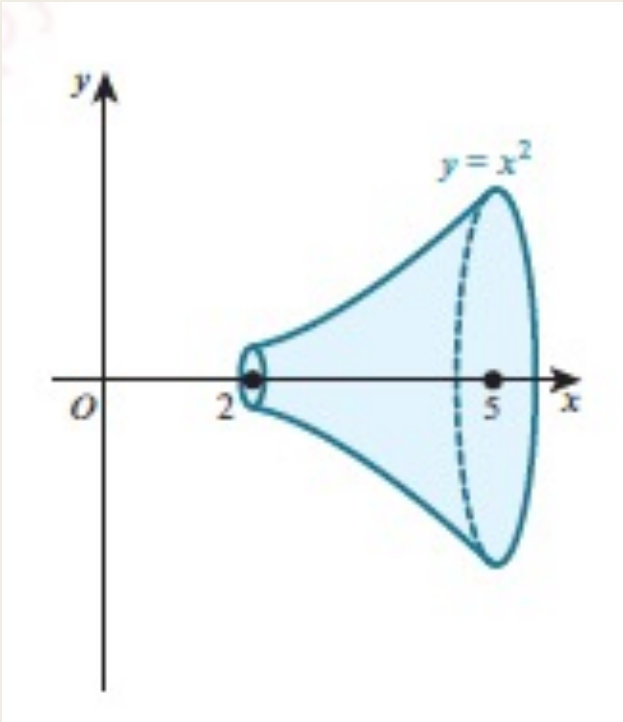
VOLUMES OF REVOLUTION

As Level

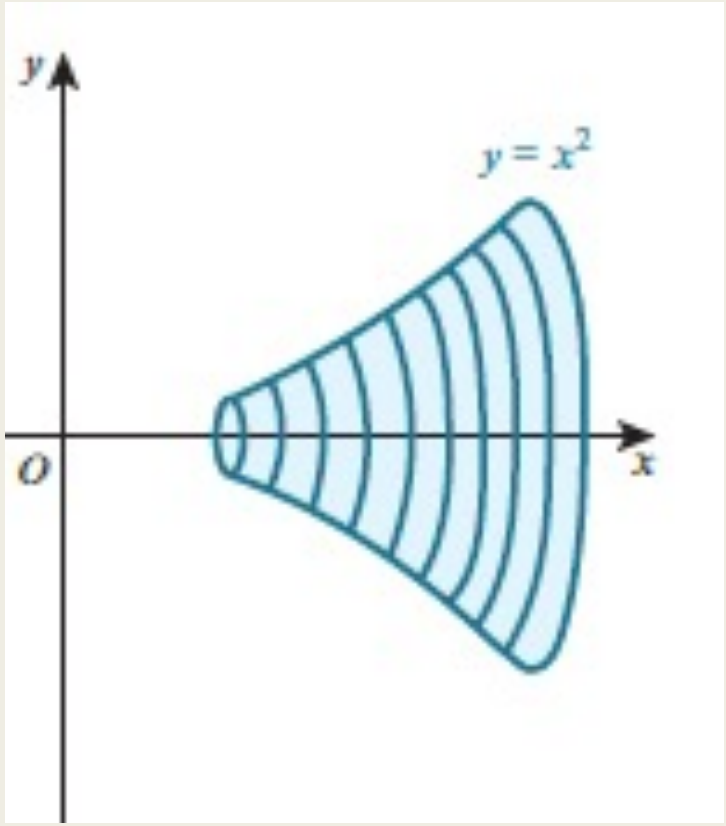
Consider the area bounded by the curve $y = x^2$, the x-axis the lines $x = 2$ and $x = 5$.



When this area is rotated about the $x - axis$ through $360°$ a **solid of revolution** is formed. The volume of this is called a **volume of revolution**.



The volume, V, obtained when the function $y = f(x)$ is rotated through $360°$ about the $x - axis$ between the boundary values $x = a$ and $x = b$ is given by the formula $V = \pi \int_a^b y^2 dx$.



Example 1:

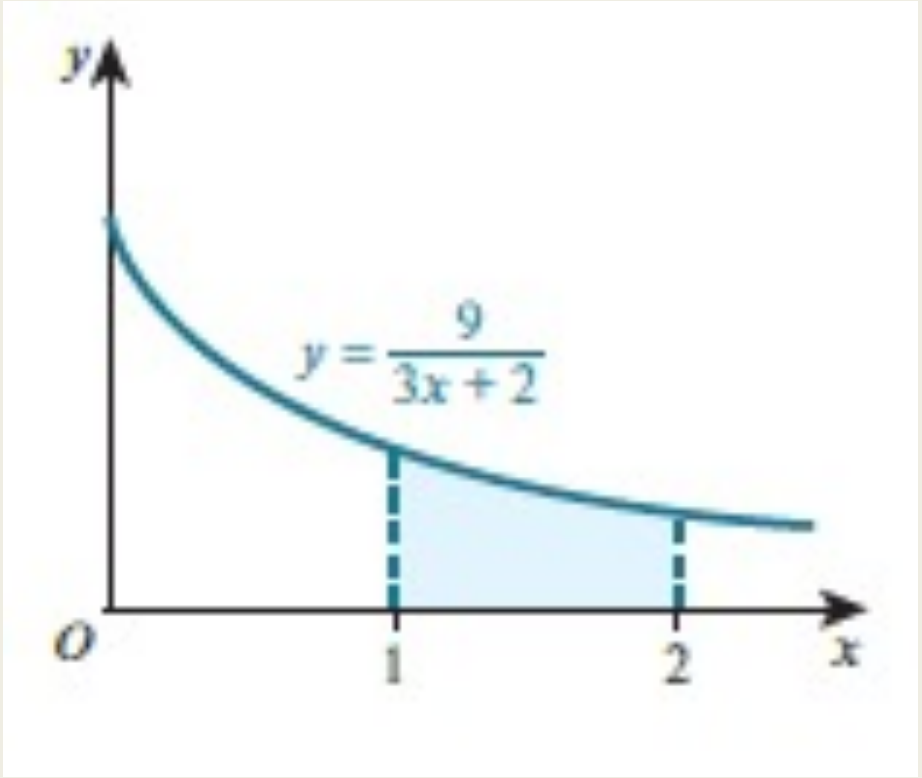
Find the volume obtained when the shaded region is rotated through $360°$ about the $x-axis$.

$$y^2 = \frac{81}{(3x+2)^2} \qquad y^2 = 81(3x+2)^{-2}$$

$$V = \pi \int_1^2 81(3x+2)^{-2}\,dx$$

$$= \pi \times \frac{81}{(-1)\times 3} \times (3x+2)^{-1}{}_1^2$$

$$= -\frac{27\pi}{(3x+2)_1}^2 \qquad = \frac{81\pi}{40}$$



Sometimes a curve is rotated about the y-axis. In this case the general rule is:

The volume, V, obtained when the function $x = f(y)$ is rotated through $360°$ about the $y - axis$ between the boundary values $y = a$ and $y = b$ is given by the formula $V = \pi \int_a^b x^2 dy$.

Example 2:

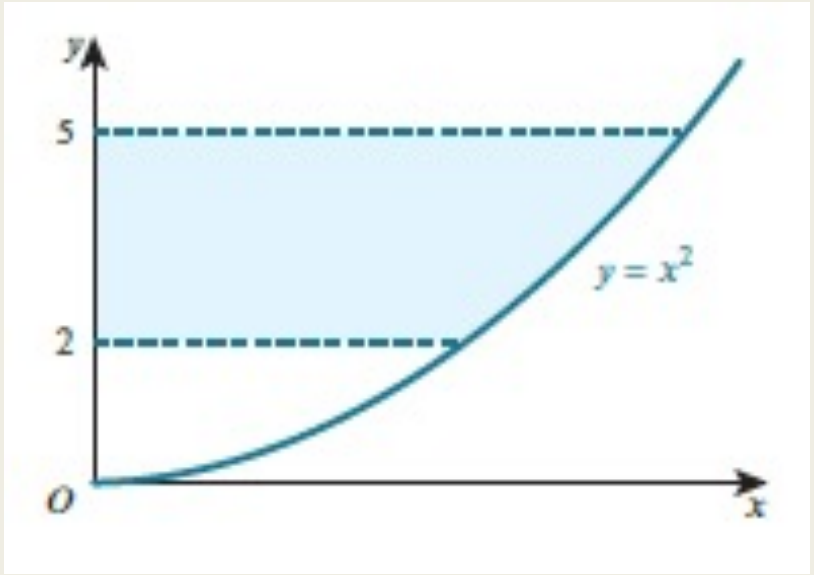
Find the volume obtained when the shaded region is rotated through $360°$ about the $y - axis$.

$$x^2 = y$$

$$V = \pi \int_2^5 y dy =$$

$$= \pi \times \frac{y^2}{2}\Big|_2^5$$

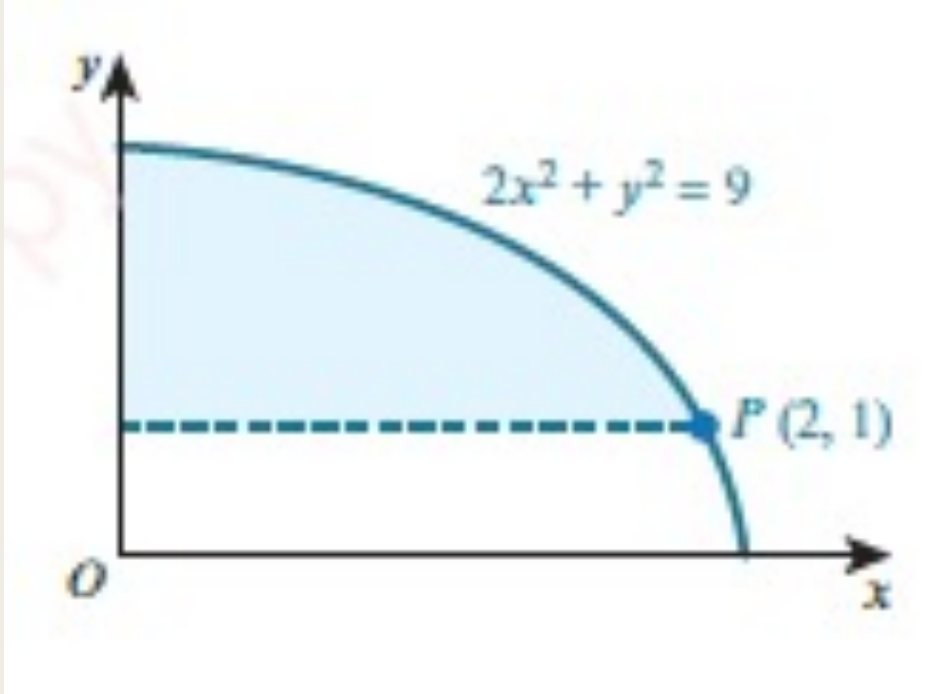
$$= \frac{21\pi}{2}$$



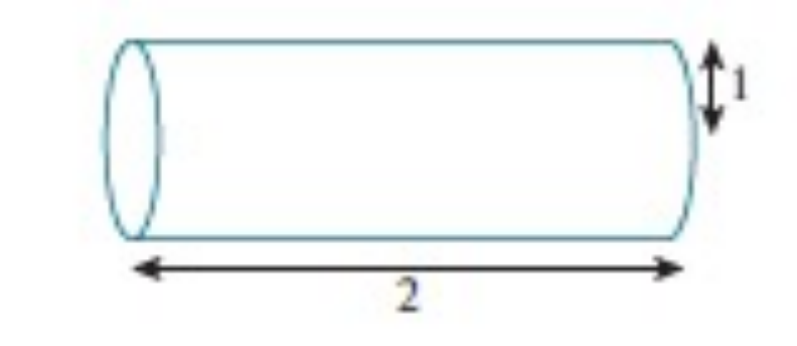
Example 3:

Find the volume obtained when the shaded region is rotated through $360°$ about the $x - axis$.

When the shaded region is rotated about the $x - axis$, a solid with a cylindrical hole is formed.



The radius of the cylindrical hole is 1 unit and the length of the hole is 2 units.



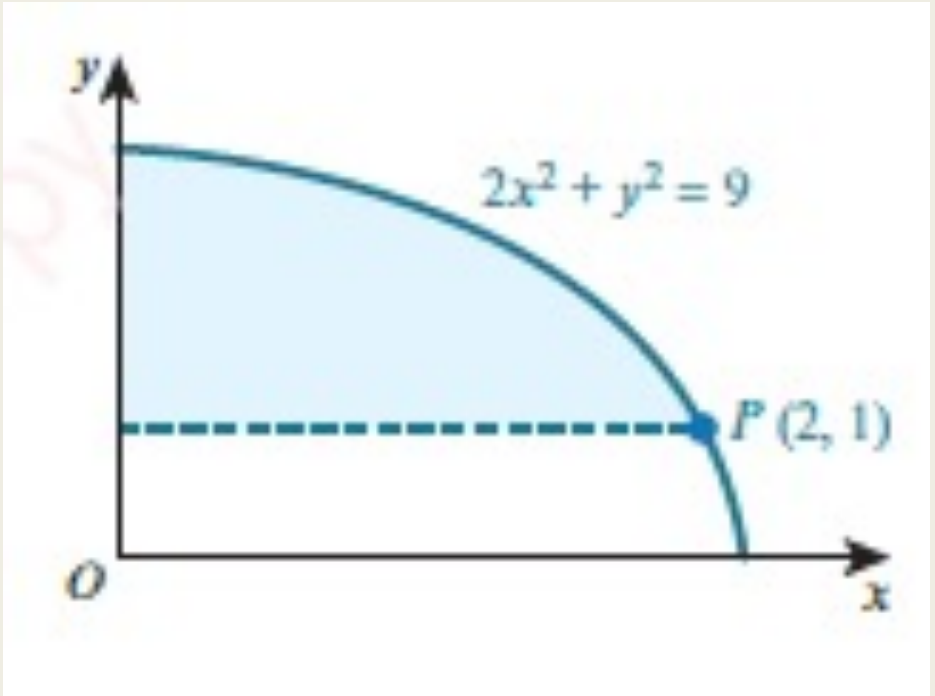
Example 3:

Find the volume obtained when the shaded region is rotated through $360°$ about the $x-axis$.

$$2x^2 + y^2 = 9$$

$$x = 0 \quad 2\times 0^2 + y^2 = 9$$

$$y = 3$$



$$V = \pi \int_1^3 y^2 dx - volume\ of\ cylinder$$

$$V = \pi \int_1^3 (9 - 2x^2) dx - \pi \times 1^2 \times 2$$

$$= \pi \left(9x - \frac{2}{3}x^3\right)_1^3 - 2\pi \qquad = \frac{32\pi}{3}$$