IGCSE
Chapter 12
Algebraic indices

Three laws of indices were introduced:

1. $a^m \times a^n = a^{m+n}$
2. $a^m \div a^n = a^{m-n}$
3. $(a^m)^n = a^{mn}$

Exercise 12.1

1 Simplify the following:

a $c^5 \times c^3$ **b** $m^4 \div m^2$

c $(b^3)^5 \div b^6$ **d** $\frac{m^4n^9}{mn^3}$

e $\frac{6a^6b^4}{3a^2b^3}$ **f** $\frac{12x^5y^7}{4x^2y^5}$

g $\frac{4u^3v^6}{8u^2y^3}$ **h** $\frac{3x^6y^5z^3}{9x^4y^2z}$

2 Simplify the following:

a $4a^2 \times 3a^3$ **b** $2a^2b \times 4a^3b^2$

c $(2p^2)^3$ **d** $(4m^2n^3)^2$

e $(5p^2)^2 \times (2p^3)^3$ **f** $(4m^2n^2) \times (2mn^3)^3$

g $\frac{(6x^2y^4)^2 \times (2xy)^3}{12xy^6y^8}$ **h** $(ab)^d \times (ab)^e$

The zero index

A term raised to the power of zero is always equal to 1.

$$a^0 = 1$$

Negative indices

A negative indicates that a number or an algebraic term is being raised to a negative power.

$$a^{-m} = \frac{1}{a^m}$$

Exercise 12.2

1 Simplify the following:

a $c^3 \times c^0$ **b** $g^{-2} \times g^3 \div g^0$

c $(p^0)^3(q^2)^{-1}$ **d** $(m^3)^3(m^{-2})^5$

2 Simplify the following:

a $\dfrac{a^{-3} \times a^5}{(a^2)^0}$ **b** $\dfrac{(r^3)^{-2}}{(p^{-2})^3}$

c $(t^3 \div t^{-5})^2$ **d** $\dfrac{m^0 \div m^{-6}}{(m^{-1})^3}$

Fractional indices

In general:

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} \text{ or } \left(\sqrt[n]{a}\right)^m$$

Example 1:

Express $(\sqrt[3]{a})^4$ in the form $a^{\frac{m}{n}}$.

$$\sqrt[3]{a} = a^{\frac{1}{3}}$$

$$(\sqrt[3]{a})^4 = \left(a^{\frac{1}{3}}\right)^4 = a^{\frac{4}{3}}$$

Example 2:

Express $b^{\frac{2}{5}}$ in the form $\left(\sqrt[n]{b}\right)^{m}$.

$$b^{\frac{1}{5}} = \sqrt[5]{b}$$

$$b^{\frac{2}{5}} = \left(b^{\frac{1}{5}}\right)^{2} = \left(\sqrt[5]{b}\right)^{2}$$

Example 3:

Simplify $\dfrac{p^{\frac{1}{2}} \times p^{\frac{1}{3}}}{p}$

$$p^{\frac{1}{2}} \times p^{\frac{1}{3}} = p^{\frac{1}{2}+\frac{1}{3}} = p^{\frac{5}{6}}$$

$$\frac{p^{\frac{5}{6}}}{p} = p^{\frac{5}{6}-1} = p^{-\frac{1}{6}} = \left(\sqrt[6]{p}\right)^{-1} = \frac{1}{\sqrt[6]{p}}$$

Exercise 12.3

1 Rewrite the following in the form $a^{\frac{m}{n}}$:

a $(\sqrt[5]{a})^3$ **b** $(\sqrt[6]{a})^2$ **c** $(\sqrt[4]{a})^4$ **d** $(\sqrt[7]{a})^3$

2 Rewrite the following in the form $(\sqrt[n]{b})^m$:

a $b^{\frac{2}{7}}$ **b** $b^{\frac{8}{3}}$ **c** $b^{-\frac{2}{5}}$ **d** $b^{-\frac{4}{3}}$

3 Simplify the following algebraic expressions, giving your answer in the form $a^{\frac{m}{n}}$:

a $a^{\frac{1}{2}} \times a^{\frac{1}{4}}$ **b** $a^{\frac{2}{5}} \times a^{-\frac{1}{4}}$ **c** $\frac{\sqrt{a}}{a^{-2}}$ **d** $\frac{\sqrt[3]{a}}{a}$

4 Simplify the following algebraic expressions, giving your answer in the form $(\sqrt[n]{b})^m$:

a $\frac{\sqrt{b} \times b^{\frac{1}{4}}}{b^{-\frac{1}{5}}}$ **b** $\frac{b^{-\frac{1}{3}} \times \sqrt[3]{b}}{b^{\frac{2}{5}} \times b}$ **c** $\frac{b^3 \times b^{-\frac{1}{3}}}{b^{-2}}$ **d** $\frac{b^{-2} \times \sqrt[3]{b}}{\sqrt{b} \times (\sqrt[3]{b})^{-1}}$

5 Simplify the following:

a $\frac{1}{3}x^{\frac{1}{2}} \div 4x^{-2}$ **b** $\frac{2}{5}y^{\frac{1}{3}} \times 5y^{-\frac{2}{3}}$ **c** $\left(2p^{-\frac{1}{4}}\right)^2 \div \frac{1}{2}p^2$ **d** $3x^{-\frac{2}{3}} \div \frac{2}{3}x^{-\frac{1}{3}}$

Student assessment 1

1 Simplify the following using indices:

a $a \times a \times a \times b \times b$ b $d \times d \times e \times e \times e \times e \times e$

2 Write the following out in full:

a m^3 b r^4

3 Simplify the following using indices:

a $a^4 \times a^3$ b $p^3 \times p^2 \times q^4 \times q^5$

c $\dfrac{b^7}{b^4}$ d $\dfrac{(e^4)^5}{e^{14}}$

4 Simplify the following:

a $r^4 \times t^0$ b $\dfrac{(a^3)^0}{b^2}$ c $\dfrac{(m^0)^5}{n^{-3}}$

5 Simplify the following:

a $\dfrac{(p^2 \times p^{-5})^2}{p^3}$ b $\dfrac{(h^{-2} \times h^{-5})^{-1}}{h^0}$

Student assessment 2

1 Rewrite the following in the form $a^{\frac{m}{n}}$:

a $(\sqrt[8]{a})$ b $(\sqrt[5]{a})^{-2}$

2 Rewrite the following in the form $(\sqrt[n]{b^m})$:

a $b^{\frac{4}{9}}$ b $b^{-\frac{2}{3}}$

3 Simplify the following algebraic expressions, giving your answer in the form $a^{\frac{m}{n}}$:

a $a^{\frac{1}{3}} \times a^{\frac{3}{2}}$ b $\dfrac{\sqrt[3]{a}}{a^{-\frac{5}{6}}} \times a^2$

4 Simplify the following algebraic expressions, giving your answer in the form $(\sqrt[n]{t})^m$:

a $\dfrac{\sqrt{t} \times t^{\frac{2}{3}}}{t^{-\frac{1}{3}}}$ b $\dfrac{\sqrt[3]{t}}{t^2 \times t^{-\frac{2}{5}}}$