BASIC RULES FOR UNCERTAINTY CALCULATIONS

Types of uncertainties:

1) An Absolute Uncertainty is denoted by the symbol "Δ" and has the same units as the quantity.
2) A Relative or Percent Uncertainty is denoted by the symbol "ε" and has no units.

To convert back and forth between the two types of uncertainties consider the following:

$$m = (3.3 \pm 0.2)\ \text{kg} = (3.3\ \text{kg} \pm 6.1\%)$$

The Absolute Uncertainty is: $\Delta m = 0.2\ \text{kg} = (6.1/100) \times 3.3\ \text{kg}$

The Relative Uncertainty is: $\varepsilon_m = 6.1\% = (0.2/3.3) \times 100\%$

Completing Uncertainty Calculations:

1) Addition and Subtraction: ADD the Absolute Uncertainties

Rule: $(A \pm \Delta A) + (B \pm \Delta B) = (A + B) \pm (\Delta A + \Delta B)$

$(A \pm \Delta A) - (B \pm \Delta B) = (A - B) \pm (\Delta A + \Delta B)$

Consider the numbers: (6.5 ± 0.5) m and (3.3 ± 0.1) m

Add: $(6.5 \pm 0.5)\ \text{m} + (3.3 \pm 0.1)\ \text{m} = (9.8 \pm 0.6)\ \text{m}$

Subtract: $(6.5 \pm 0.5)\ \text{m} - (3.3 \pm 0.1)\ \text{m} = (3.2 \pm 0.6)\ \text{m}$

2) Multiplication and Division: ADD the Relative Uncertainties

Rule: $(A \pm \varepsilon_A) \times (B \pm \varepsilon_B) = (A \times B) \pm (\varepsilon_A + \varepsilon_B)$

$(A \pm \varepsilon_A) / (B \pm \varepsilon_B) = (A / B) \pm (\varepsilon_A + \varepsilon_B)$

Consider the numbers: $(5.0\ \text{m} \pm 4.0\%)$ and $(3.0\ \text{s} \pm 3.3\%)$

Multiply: $(5.0\ \text{m} \pm 4.0\%) \times (3.0\ \text{s} \pm 3.3\%) = (15.0\ \text{m}\bullet\text{s} \pm 7.3\%)$

Divide: $(5.0\ \text{m} \pm 4.0\%) / (3.0\ \text{s} \pm 3.3\%) = (1.7\ \text{m/s} \pm 7.3\%)$

3) For a number raised to a power, fractional or not, the rule is simply to MULTIPLY the Relative Uncertainty by the power.

Rule: $(A \pm \varepsilon_A)^n = (A^n \pm n\varepsilon_A)$

Consider the number: $(2.0\ \text{m} \pm 1.0\%)$

Cube: $(2.0\ \text{m} \pm 1.0\%)^3 = (8.0\ \text{m}^3 \pm 3.0\%)$

Square Root: $(2.0\ \text{m} \pm 1.0\%)^{1/2} = (1.4\ \text{m}^{1/2} \pm 0.5\%)$

4) For multiplying a number by a constant there are two different rules depending on which type of uncertainty you are working with at the time.

Rule - Absolute Uncertainty: $c(A \pm \Delta A) = cA \pm c(\Delta A)$

Consider: $1.5(2.0 \pm 0.2)$ m = (3.0 ± 0.3) m
Note that the Absolute Uncertainty **is** multiplied by the constant.

Rule - Relative Uncertainty: $c(A \pm \varepsilon_A) = cA \pm \varepsilon_A$

Consider: $1.5(2.0 \text{ m} \pm 1.0\%) = (3.0 \text{ m} \pm 1.0\%)$
Note that the Relative Uncertainty **is not** multiplied by the constant.

Comparing Values:

1) When comparing two values you should complete a CONSISTENCY CHECK. This is the preferred way to compare values which have uncertainties.

Rule: If the following inequality is true you may say that the values you are comparing are consistent with each other within experimental uncertainty, otherwise the values are inconsistent.

$$|A - B| \leq |\Delta A + \Delta B|$$

Consider the numbers: (3.3 ± 0.2) m and (3.1 ± 0.1) m
Consistency check: $|3.3 - 3.1|$ m $\leq |0.2 + 0.1|$ m
$|0.2|$ m $\leq |0.3|$ m $\therefore$ Consistent Values

Note that the difference of the two quantities is less than or equal to the sum of the Absolute Uncertainties. You cannot complete a consistency check using Relative Uncertainties.
Sometimes you will be comparing an experimental value which has an uncertainty to a theoretical value which may not have an uncertainty. In this case the above calculation still holds.

Compare the numbers: $g = 9.87 \text{ m/s}^2 \pm 0.09 \text{ m/s}^2$ Experimental value
$g = 9.81 \text{ m/s}^2$ Theoretical Value
Consistency check: $|9.87 - 9.81| \text{ m/s}^2 \leq |0.09| \text{ m/s}^2$
$|0.06| \text{ m/s}^2 \leq |0.09| \text{ m/s}^2$ Consistent

2) When comparing two values which don't have uncertainties you may then calculate a Percent Difference as follows:

$$\%Diff = \frac{|Theor.Value - Exp.Value|}{Theor.Vlaue} x100\%$$