CAIE Physics A-level

Topic 12: Motion in a Circle

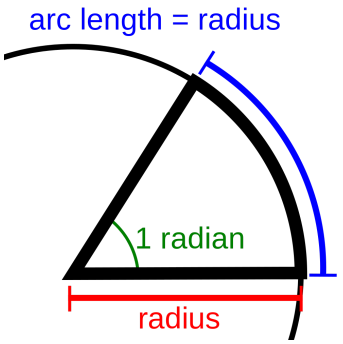
Notes

12 - Motion in a Circle (A-level only)

12.1 - Kinematics of Uniform Circular Motion

Angles can be measured in units called **radians**. One radian is defined as the angle in the sector of a circle when the arc length of that sector is equal to the radius of the circle, as shown in the diagram below.

Considering a complete circle, its arc length is $2\pi r$, dividing this by r, you get 2π which is the angle in radians of a full circle. From this you can convert any angle from **degrees to radians** by multiplying by $\frac{\pi}{180}$, and from **radians to degrees** by multiplying by $\frac{180}{\pi}$.



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Angular displacement (θ) is the **angle turned through** by an object in any given direction **in radians**.

Angular speed (ω) is the angle an object moves through per unit time. It can be found by dividing the object's linear speed (v) by the radius of the circular path it is travelling in (r), or by dividing the angle in a circle in radians (2π) by the object's time period (T).

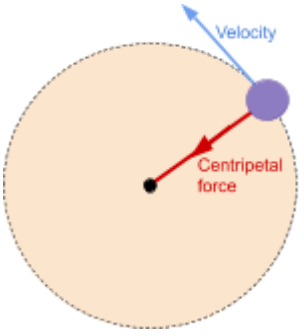
$$\omega = \frac{v}{r} = \frac{2\pi}{T} = 2\pi f \quad \text{as } f = \frac{1}{T}$$

You can calculate an object's **linear speed (v)** by finding the product of its angular speed (ω) and the radius of its path (r) as shown below:

$$v = r\omega$$

12.2 - Centripetal Acceleration and Centripetal Force

An object moving in a circular path at constant speed has a constantly changing velocity as velocity has both magnitude and direction, therefore the object must be **accelerating** (this is known as centripetal acceleration). We know from Newton's first law that to accelerate, an object must experience a resultant force, therefore an object moving in a circle must experience a force. This is known as the **centripetal force**, and it always acts **perpendicular** to the motion of the object (towards the centre of the circle).



Centripetal acceleration (a) can be found using the formula below:

$$a = \frac{v^2}{r} = \omega^2 r$$

Where v is linear speed, r is the radius of the path and ω is the angular speed.

Centripetal acceleration causes circular motion with a constant angular speed.

Using Newton's second law, $F = ma$, we can derive the formula for **centripetal force (F)** from the formula above.

$$F = \frac{mv^2}{r} = m\omega^2 r$$

Where m is the mass of the object, v is linear speed, r is the radius of the path and ω is the angular speed.