IGCSE
Chapter 14
Linear programming

Graphing more than one inequality

Several inequalities can be graphed on the same set of axes. If the regions which satisfy each inequality are left unshaded, then a solution can be found which satisfies all the inequalities.

Example 6: A region R, contains points whose coordinates satisfy the following inequalities.

$x \geq -4$ $y > -1$ $x + 2y \leq 2$

On a graph draw suitable lines and label the region R.

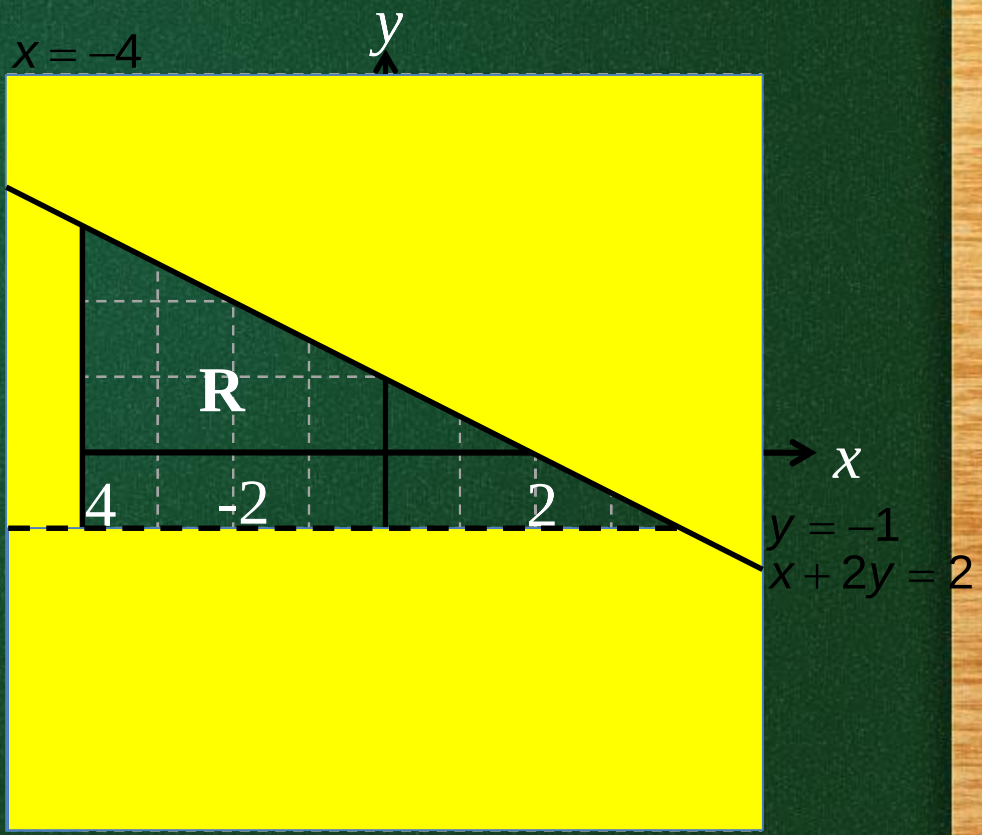
First show the region $x \geq -4$ on the graph (remember to shade the unwanted region).

Next show $y > -1$ (remember to shade the unwanted region).

Then show $x + 2y \leq 2$ (remember to shade the unwanted region).

$x + 2y = 2$

x	0	2
y	1	0



Exercise 14.3

On the same pair of axes, plot the following inequalities and leave unshaded the region which satisfies all of them simultaneously.

1. $y \leqslant x$ $\quad y > 1$ $\quad x \leqslant 5$
2. $x + y \leqslant 6$ $\quad y < x$ $\quad y \geqslant 1$
3. $y \geqslant 3x$ $\quad y \leqslant 5$ $\quad x + y > 4$
4. $2y \geqslant x + 4$ $\quad y \leqslant 2x + 2$ $\quad y < 4$ $\quad x \leqslant 3$

Linear programming

Linear programming is a way of finding a number of possible solutions to a problem given a number of constraints. But it is more than this – it is also a method for minimising a linear function in two (or more) variables.

Example 1: $x \geq 3$ $y > 2$ $x + y \leq 8$

a Show the points with integer coordinates that satisfy all three inequalities.

Remember to shade the **unwanted** regions.

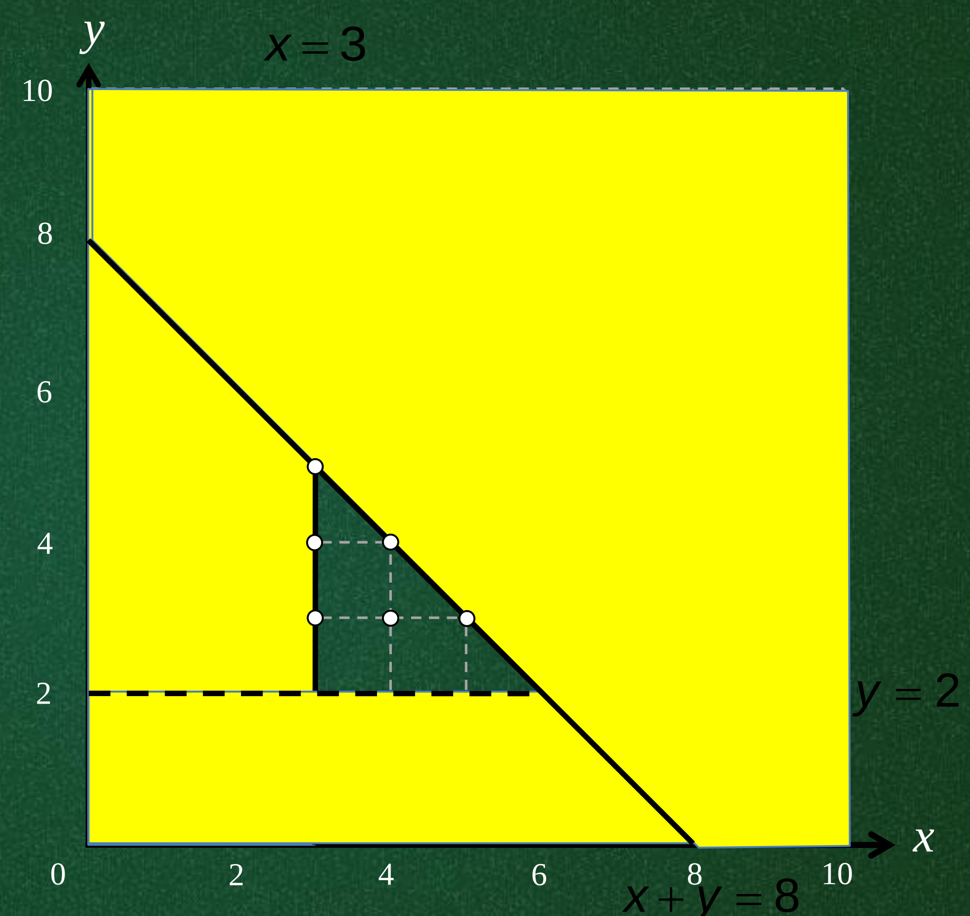
First show $x \geq 3$.

Next show $y > 2$.

Then show $x + y \leq 8$.

$x + y = 8$

x	0	8
y	8	0

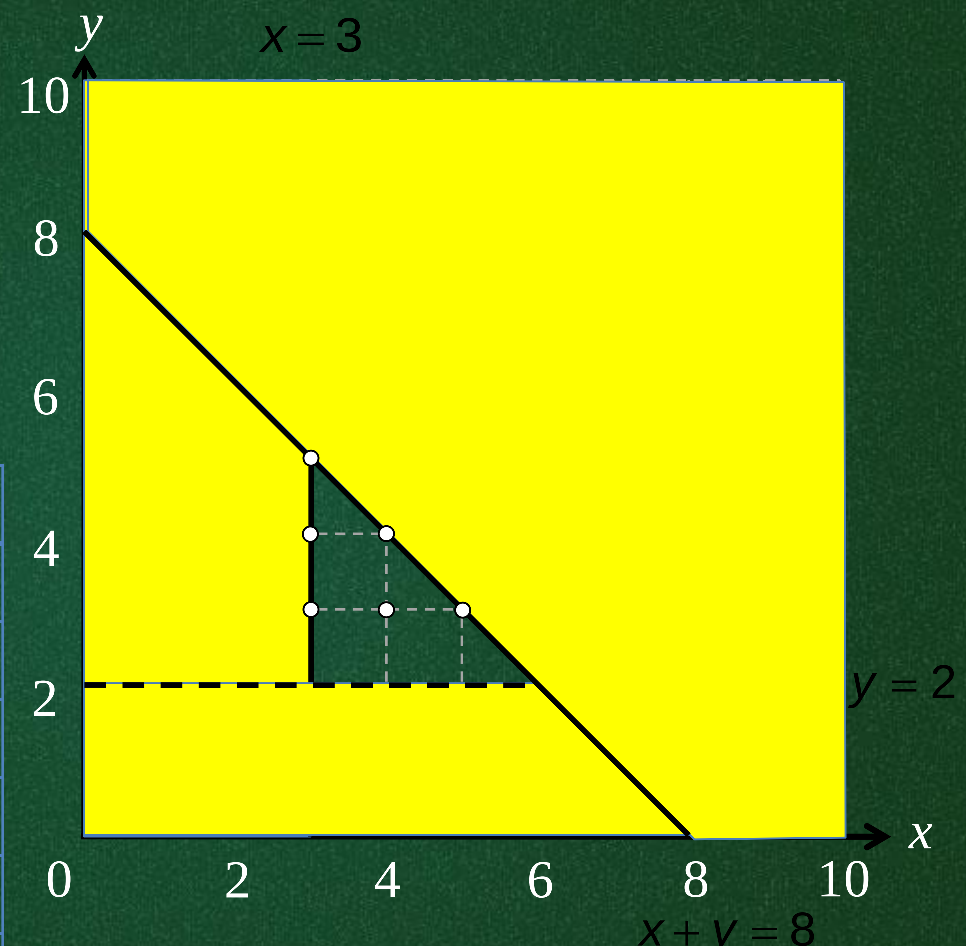


Now show the points with integer coordinates that satisfy all three inequalities on the diagram.

b For the points in part **a**, find the maximum value of $2x + y$.

Put the points into a table and calculate the value of $2x + y$ at each point.

x	y	$2x + y$
3	3	6 + 3 = 9
4	3	8 + 3 = 11
5	3	10 + 3 = 13
3	4	6 + 4 = 10
4	4	8 + 4 = 12
3	5	6 + 5 = 11



The maximum value of $2x + y$ is 13.
(The maximum occurs at the point (5, 3).)

It is important to note that the maximum or minimum values always occur at/near the corners of the required region

Example 2: $x \geq 1$ $y \geq x$ $4x + 5y \leq 40$

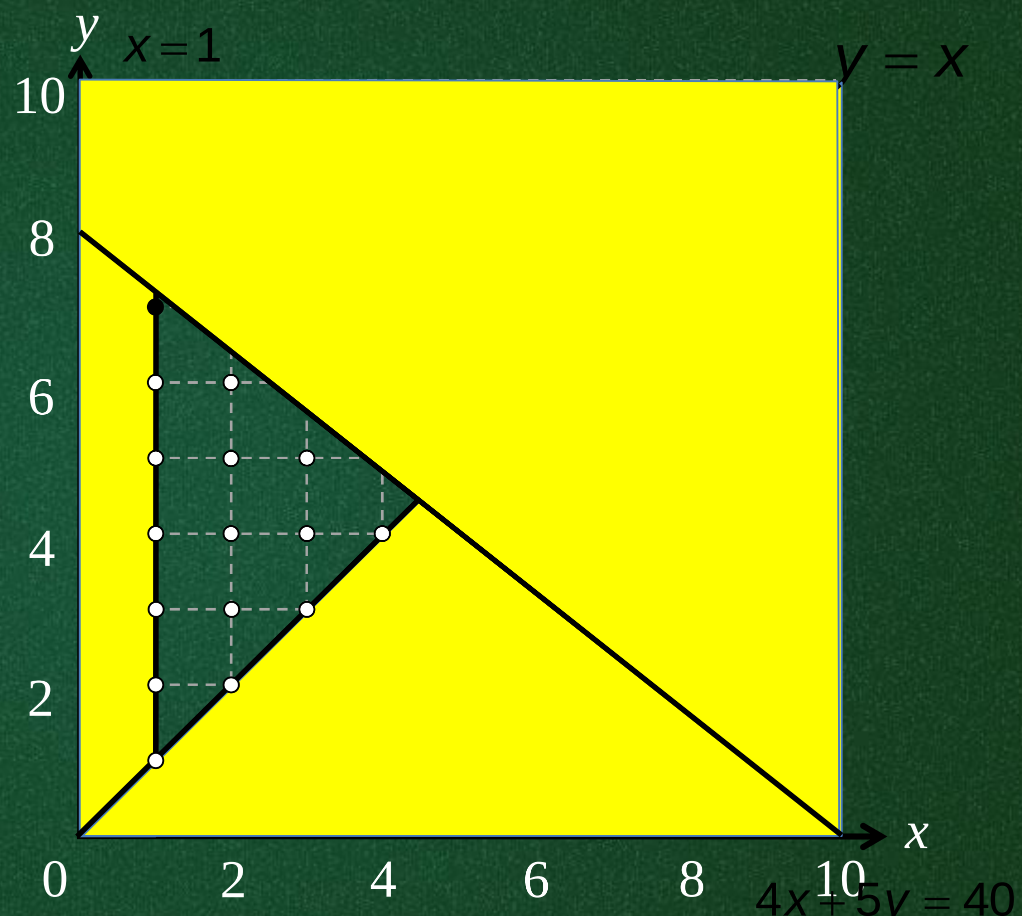
a Show the points with integer coordinates that satisfy all three inequalities.

Remember to shade the **unwanted** regions.

First show $x \geq 1$.
Next show $y \geq x$.
Then show $4x + 5y \leq 40$.

$4x + 5y = 40$

x	0	10
y	8	0

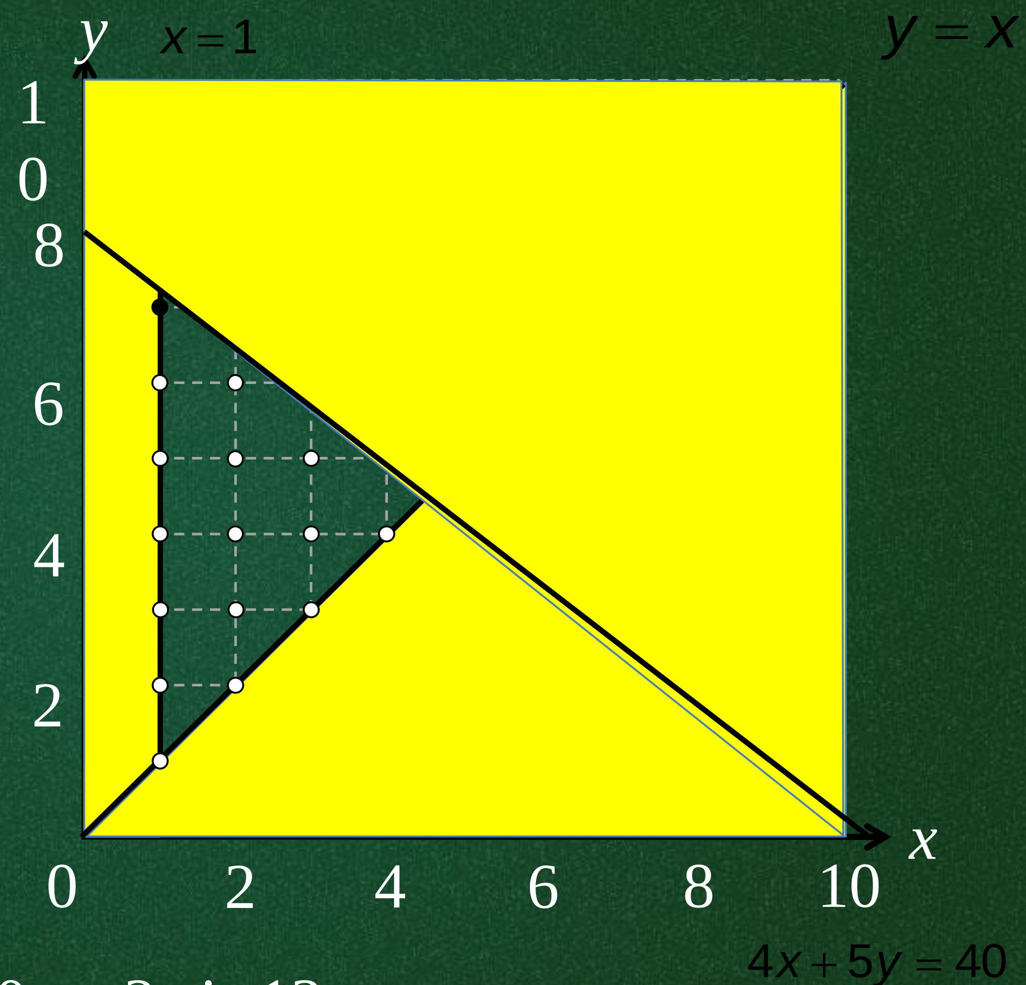


Now show the points with integer coordinates that satisfy all three inequalities on the diagram.

b For the points in part **a**, find the minimum value of $10x + 3y$.

Put the 'corner' points into a table and calculate the value of $10x + 3y$ at each point.

x	y	$10x + 3y$
1	7	10 + 21 = 31
1	1	10 + 3 = 13
4	4	40 + 12 = 52



The minimum value of $10x + 3y$ is 13.

(The minimum occurs at the point (1, 1).)

Example 3: Zoe buys *x* lemons and y oranges.

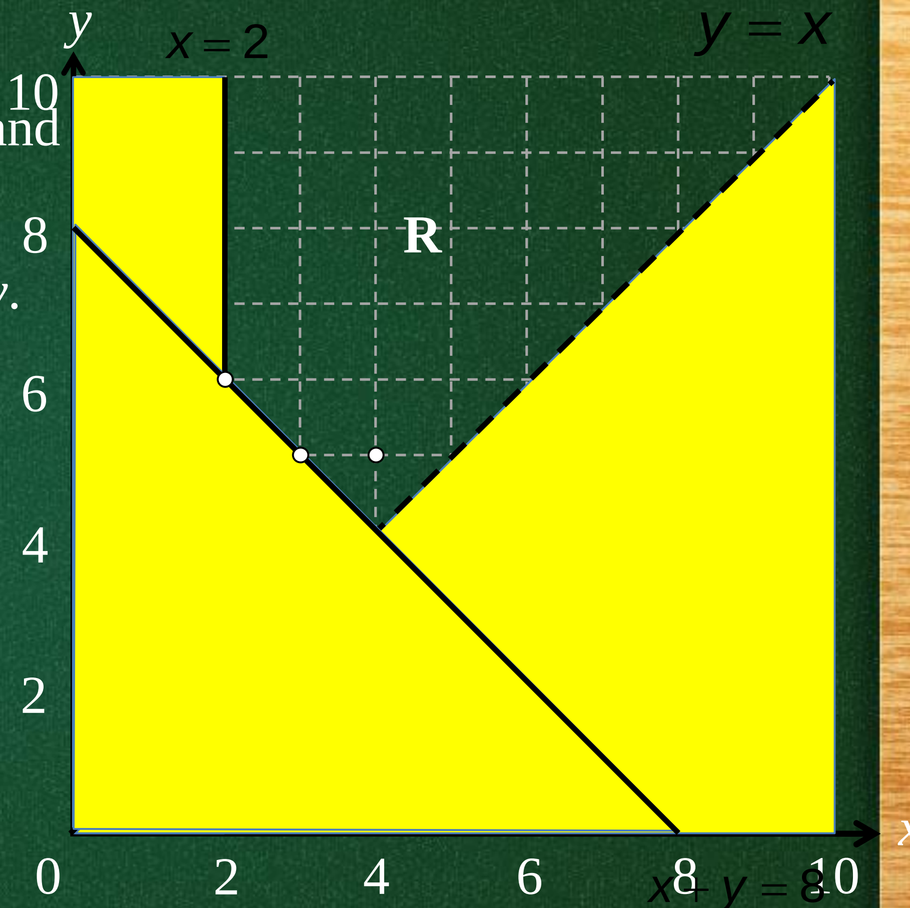
She must buy at least 2 lemons.
She must buy more oranges than lemons.
She must buy a total of at least 8 lemons and oranges.

a Write down three inequalities in *x* and *y*.

$x \geq 2$ $y > x$ $x + y \geq 8$

b On the graph show the region R that represents these 3 inequalities.

c A lemon costs $1 and an orange costs $2. Find the smallest amount of money that she spends.



At (2, 6) Cost = 2 × 1 + 6 × 2 = $14
At (3, 5) Cost = 3 × 1 + 5 × 2 = $13
At (4, 5) Cost = 4 × 1 + 5 × 2 = $14

Remember that the maximum or minimum values always occur at/near the corners of the required region.

Exercise 14.4

In the following questions draw both axes numbered from 0 to 12. For each question:

a write an inequality for each statement,
b graph the inequalities, leaving the region which satisfies the inequalities unshaded,
c using your graph, state one solution which satisfies all the inequalities simultaneously.

1 A taxi firm has at its disposal one morning a car and a minibus for hire. During the morning, it makes x car trips and y minibus trips.
   - It makes at least five car trips.
   - It makes between two and eight minibus trips.
   - The total number of car and minibus trips does not exceed 12.

2 A woman is baking bread and cakes. She makes p loaves and q cakes. She bakes at least five loaves and at least two cakes but no more than ten loaves and cakes altogether.

3 A couple are buying curtains for their house. They buy m long curtains and n short curtains. They buy at least two long curtains. They also buy at least twice as many short curtains as long curtains. A maximum of 11 curtains are bought altogether.

4 A shop sells large and small oranges. A girl buys L large oranges and S small oranges. She buys at least three but fewer than nine large oranges. She also buys fewer than six small oranges. The maximum number of oranges she needs to buy is 10.