Subsets

If all the elements of one set X are also elements of another set Y, then X is said to be a **subset** of Y.

This is written as $X \subseteq Y$.

If a set A is empty then this is called the **empty set** and it is represented by the symbol $\varnothing$. Therefore $A = \varnothing$. The empty set is a subset of all sets.

Example 5:

Three girls, Winnie, Natalie and Emma, form a set *A*.

A={Winnie, Natalie, Emma}

All the possible subsets of *A* are given below:

B={Winnie, Natalie, Emma}

C= {Winnie, Natalie}

D={Winnie, Emma}

E={Natalie, Emma}

F={Winnie}

G={Natalie}

H={Emma}

I=∅

A={Winnie, Natalie, Emma}

B={Winnie, Natalie, Emma}

C= {Winnie, Natalie}

D={Winnie, Emma}

E={Natalie, Emma}

F={Winnie}

G={Natalie}

H={Emma}

I=∅

The sets B and I above are considered as subsets of A.

$B \subseteq A$ and $I \subseteq A$.

However, sets C, D, E, F, G and H are considered **proper subsets** of A.

$C \subset A$ and $D \subset A$ …

Similarly, $G \subseteq |H$ implies that G is not a subset of H.

$G \subset |H$ implies that G is not a proper subset of H.

Example 6: $A=\{1,2,3,4,5,6,7,8,9,10\}$

a. List subset B {even numbers}

$B=\{2,4,6,8,10\}$

b. List subset C {prime numbers}

$C=\{2,3,5,7\}$

Exercise 10.2

1 P = {whole numbers less than 30}
- a List the subset Q {even numbers}.
- b List the subset R {odd numbers}.
- c List the subset S {prime numbers}.
- d List the subset T {square numbers}.
- e List the subset U {triangle numbers}.

2 A = {whole numbers between 50 and 70}
- a List the subset B {multiples of 5}.
- b List the subset C {multiples of 3}.
- c List the subset D {square numbers}.

3 J = {p, q, r}
- a List all the subsets of J.
- b List all the proper subsets of J.

4 State whether each of the following statements is true or false:
- a {Algeria, Mozambique} $\subseteq$ {countries in Africa}
- b {mango, banana} $\subseteq$ {fruit}
- c $\{1, 2, 3, 4\} \subseteq \{1, 2, 3, 4\}$
- d $\{1, 2, 3, 4\} \subset \{1, 2, 3, 4\}$
- e {volleyball, basketball} $\not\subseteq$ {team sport}
- f $\{4, 6, 8, 10\} \not\subset \{4, 6, 8, 10\}$
- g {potatoes, carrots} $\subseteq$ {vegetables}
- h $\{12, 13, 14, 15\} \not\subset$ {whole numbers}

The universal set

The **universal set** (ξ) for any particular problem is the set which contains all the possible elements for that problem.

The **complement** of a set A is the set of elements which are in ξ but not in A. The complement of A is identified as A'. Notice that $\xi' = \emptyset$ and $\emptyset' = \xi$.

Example 7:

If $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $A = \{1, 2, 3, 4, 5\}$ what set is represented by A'?

A' consists of those elements in ξ which are not in A.

Therefore $A' = \{6, 7, 8, 9, 10\}$.

Example 8:

If ξ = {all 3D shapes} and P = {**prisms**} what set is represented by P'?

P' = {all 3D shapes except prisms}.