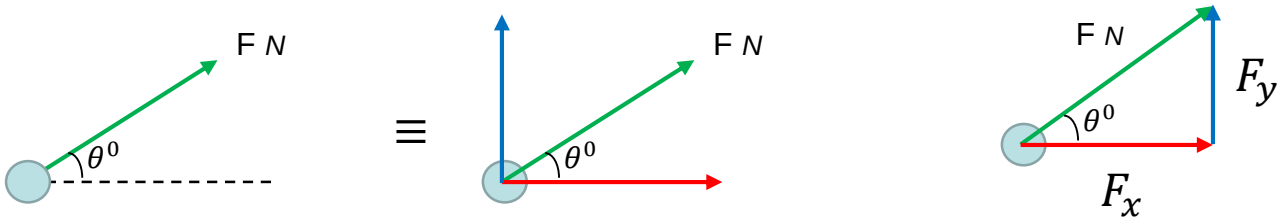
Resolving forces into components
/Хүчийг байгуулагч хүчээр илэрхийлэх/

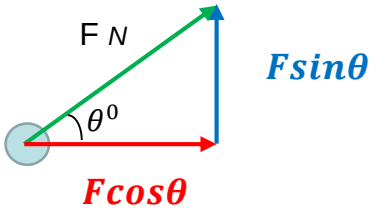
Resolving a force is about replacing a force by two forces at right angles to one another that would have the same effect as the single force.
The forces that replace the single force are called components of that force.

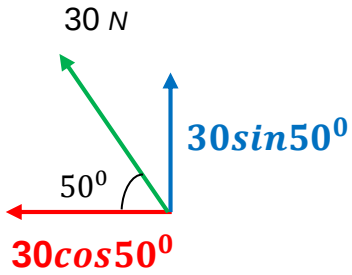


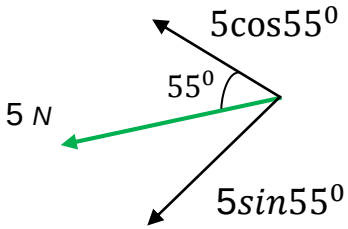
$$\sin\theta = \frac{\textbf{opposite}}{\textbf{hypotenuse}} \qquad \cos\theta = \frac{\textbf{adjacent}}{\textbf{hypotenuse}}$$

$$\sin\theta = \frac{F_y}{F} \qquad F_y = F\sin\theta$$

$$\cos\theta = \frac{F_x}{F} \qquad F_x = F\cos\theta$$







Example1

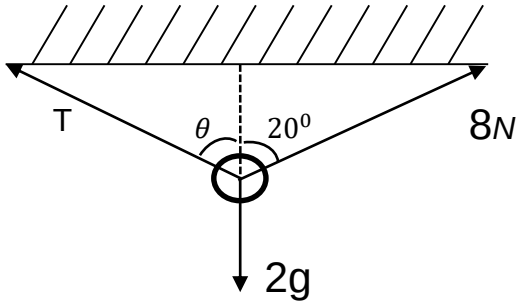
A Lightshade of mass 2 kg is hung from the ceiling by two strings. One is fixed with tension 8N at 20^0 to the vertical. The other is fixed with tension T *N* at an angle θ to the vertical.

a) By modelling the lightshade as a particle, draw a force diagram for this situation.

b) Resolve horizontally to find a value for T$sin\theta$ and resolve vertically to find a value for T$cos\theta$

c) Hence, find the value of T and θ

a)



b) $\leftarrow \quad Tsin\theta = 8sin20^0 = \mathbf{2.74}$

$\uparrow \quad Tcos\theta + 8cos20^0 = 20$

$Tcos\theta = 20 - 8cos20^0 = \mathbf{12.5}$

c) $tan\theta = \dfrac{8sin\theta}{20 - 8cos20^0} \quad \Rightarrow \quad \boldsymbol{\theta = 12.4^0}$

$T^2 = (8sin\theta)^2 + (2 - 8cos20^0)^2 \quad \Rightarrow \quad \boldsymbol{T = 12.8\ N}$

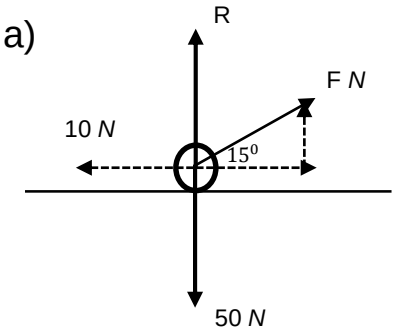
Example2

A box of weight 50*N* is being dragged at constant velocity along a horizontal road by a force, F, acting at 15^0above the horizontal. It experiences friction of 10 *N.*

a) Draw the force diagram for this situation.

b) Find F and the normal contact force.

a)



b) $\rightarrow \quad Fcos15^0 - 10 = 0 \quad \Longrightarrow Fcos15^0 = 10$

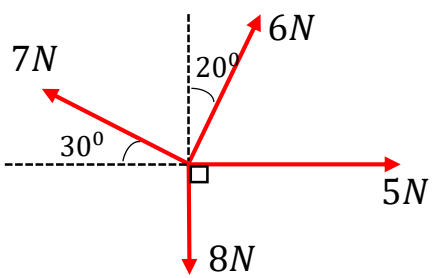
$\uparrow \quad Fsin15^0 + R - 50 = 0 \quad \Longrightarrow Fsin15^0 + R = 50$

$$F = \frac{10}{cos15^0} = \mathbf{10.4\ N}$$

$$R = \mathbf{47.3\ N}$$

Example3

Find the resultant force acting on a particle by these forces and find the angle that it makes with the 5N force.



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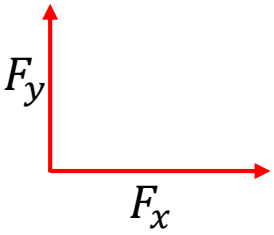

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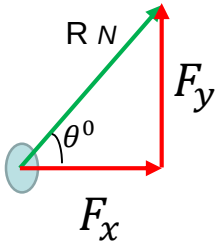

resultant force

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$\rightarrow \quad 5 + 6sin20^0 - 7cos30^0 = 0.9899N$

$\uparrow \quad 6cos20^0 + 7sin30^0 - 8 = 1.1381N$

$$R = \sqrt{(0.9899)^2 + (1.1381)^2} = \mathbf{1.51N}$$

$$tan\theta = \frac{1.1381}{0.9899} \qquad \boldsymbol{\theta = 49.0^0}$$

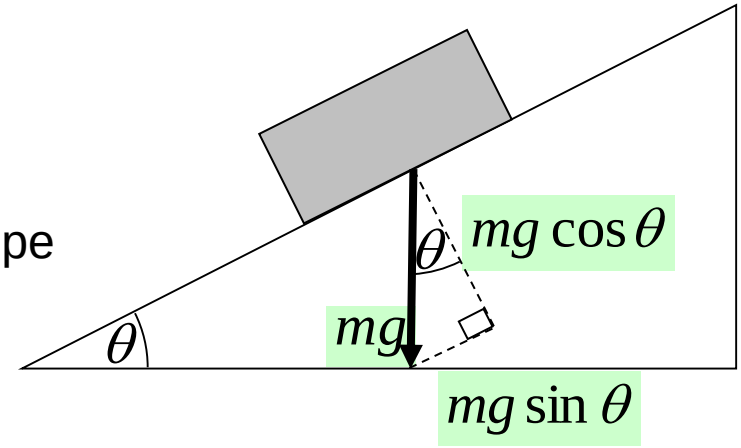
Resolving forces at other angles

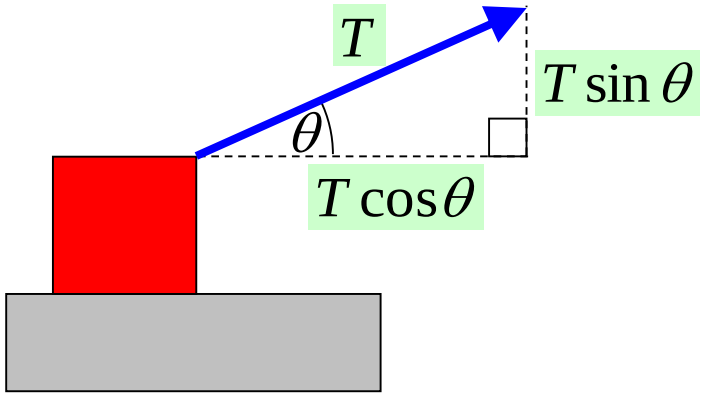
Consider a block on a slope:

The **weight** of the block has:

a component acting **perpendicular** to the slope

a component acting **parallel** to the slope





When you intend to resolve forces, you can use the following notation:

$R(\nearrow)$ resolve parallel to the slope

$R(\searrow)$ resolve perpendicular to the slope

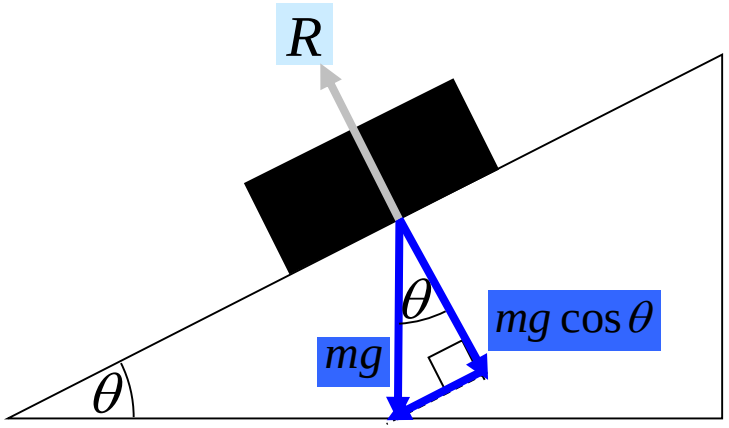
$R(\rightarrow)$ resolve horizontally

$R(\uparrow)$ resolve vertically

Normal contact(reaction) force

The normal reaction is the force which acts perpendicular to a surface when an object is contact with the surface. This must be equal to the resultant force an object is applying to the surface, as the object is not accelerating.

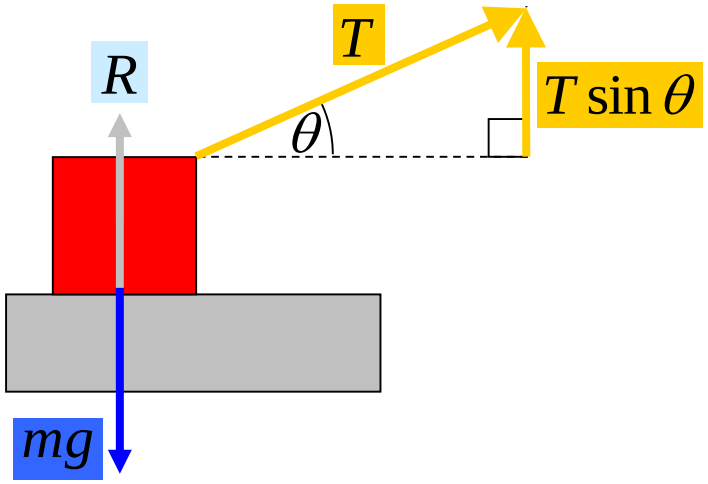
Eg a block at rest on a slope



$R(\nwarrow)$ $R - mg\cos\theta = 0$

$\Rightarrow R = mg\cos\theta$

Eg a block at rest on a surface, despite being pulled by a string

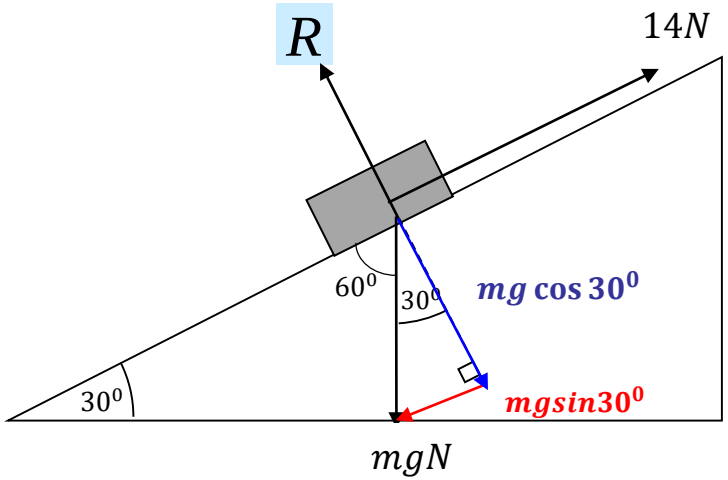


$R(\uparrow)$ $R + T\sin\theta - mg = 0$

$\Rightarrow R = mg - T\sin\theta$

Example4

An object of mass m kg is held in static equilibrium on an inclined plane by a force of 14 N, which acts up and parallel to the plane as shown in the diagram. Find the mass m and magnitude of the normal contact force R, correct to 2 decimal places. (In this question it is assumed $g = 10ms^{-2}$)



↗ $14 - mgsin30^0 = 0$

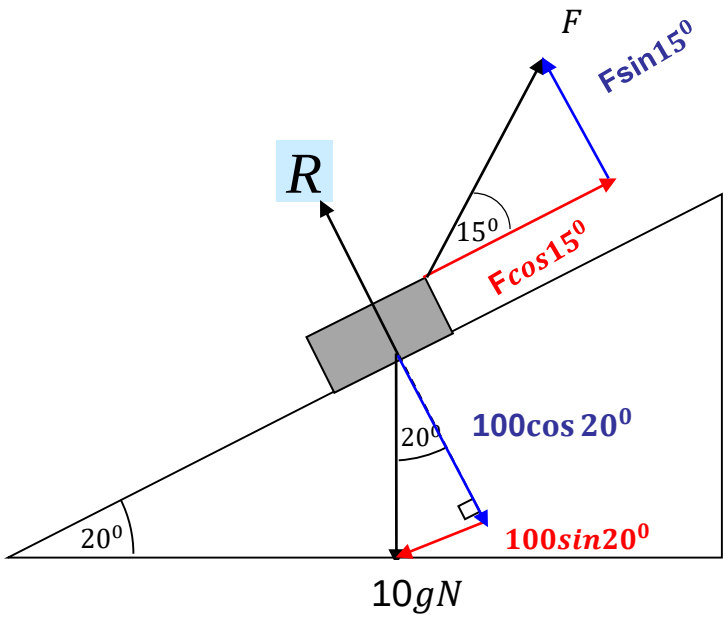
$$m = \frac{14}{10sin30^0} = \frac{20}{7} = \mathbf{2.86\ kg}$$

↖ $R - mgcos30^0 = 0$

$$R = \frac{20}{7} \cdot 10 \cdot cos30^0 = 14\sqrt{3} = \mathbf{24.2}$$

Example5

A block of mass 10kg is held in equilibrium on a slope at an angle of 20^0 to the horizontal by a force F acting at 15^0 above the slope. Find F and the normal contact force



↗ $\mathrm{F}cos15^0 - 100sin20^0 = 0$

$$F = \frac{100sin20^0}{cos15^0} = \mathbf{35.4\ N}$$

↖ $R + Fsin15^0 - 100cos20^0 = 0$

$$R = 100cos20^0 - Fsin15^0 = \mathbf{84.8N}$$