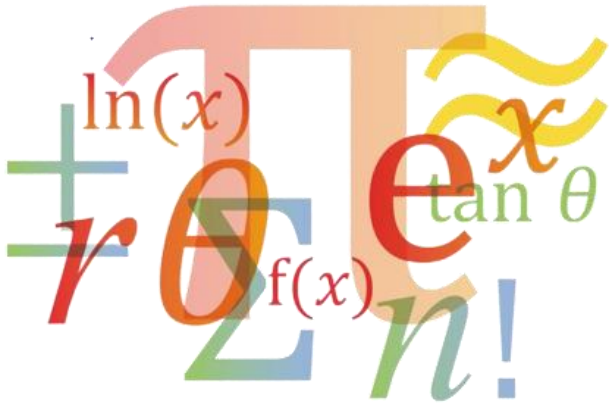
Cambridge Assessment International Education

Chapter10:

Constructions

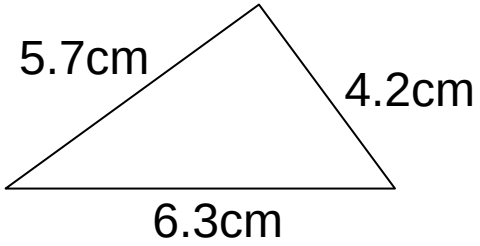


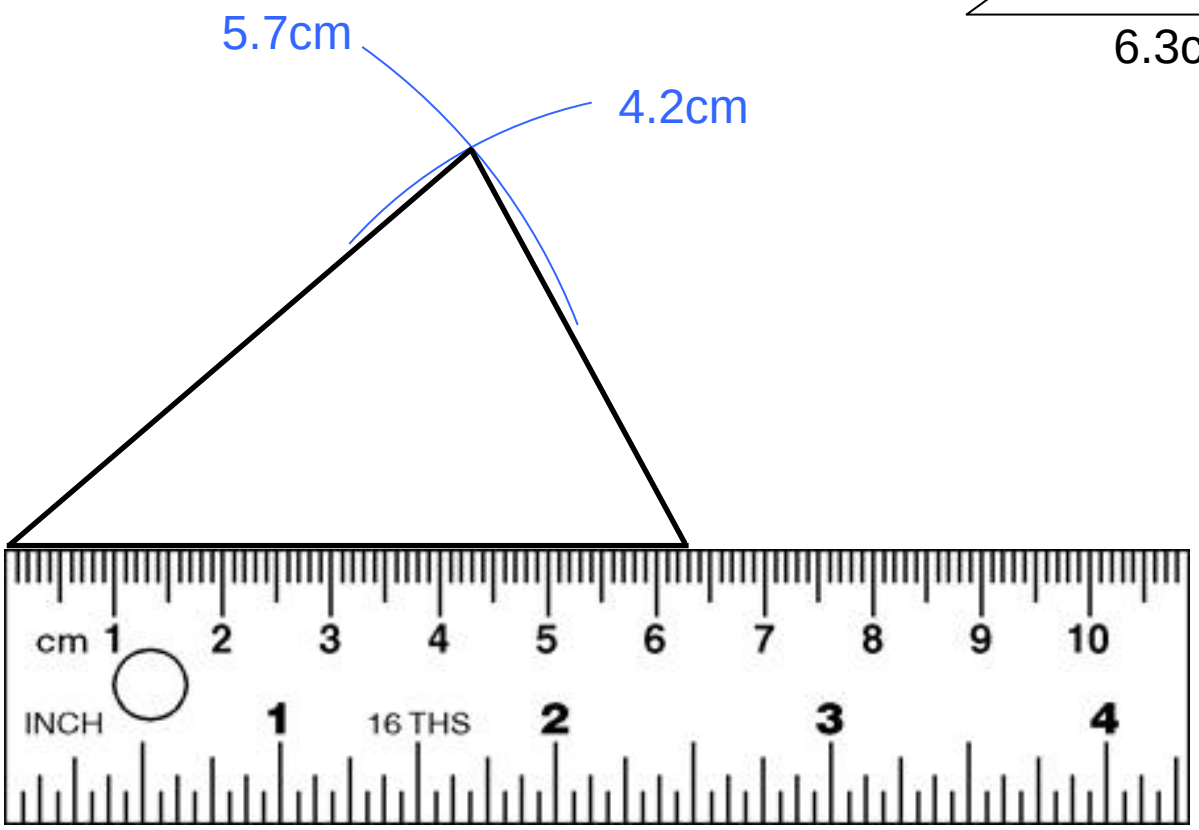
New word

1. Compass - гортиг
2. ruler - шугам
3. construct - байгуулах
4. arc -нум
5. midpoint –дундаж цэг

Ruler and compass constructions

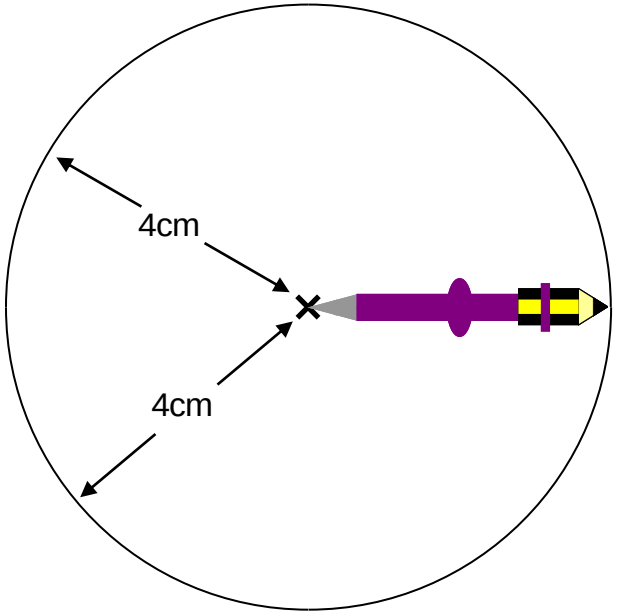
Eg use ruler & compass to construct this triangle accurately:





Circles, arcs and compasses

What is a circle? A circle is the set of all points an equal distance from its centre



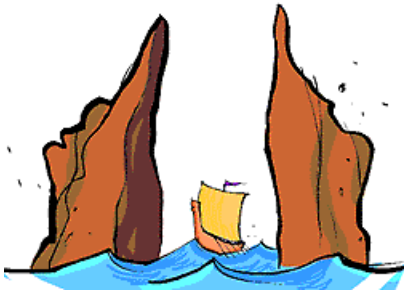
A compass is a mathematical instrument for drawing **circles** and **arcs** (part of a circle).

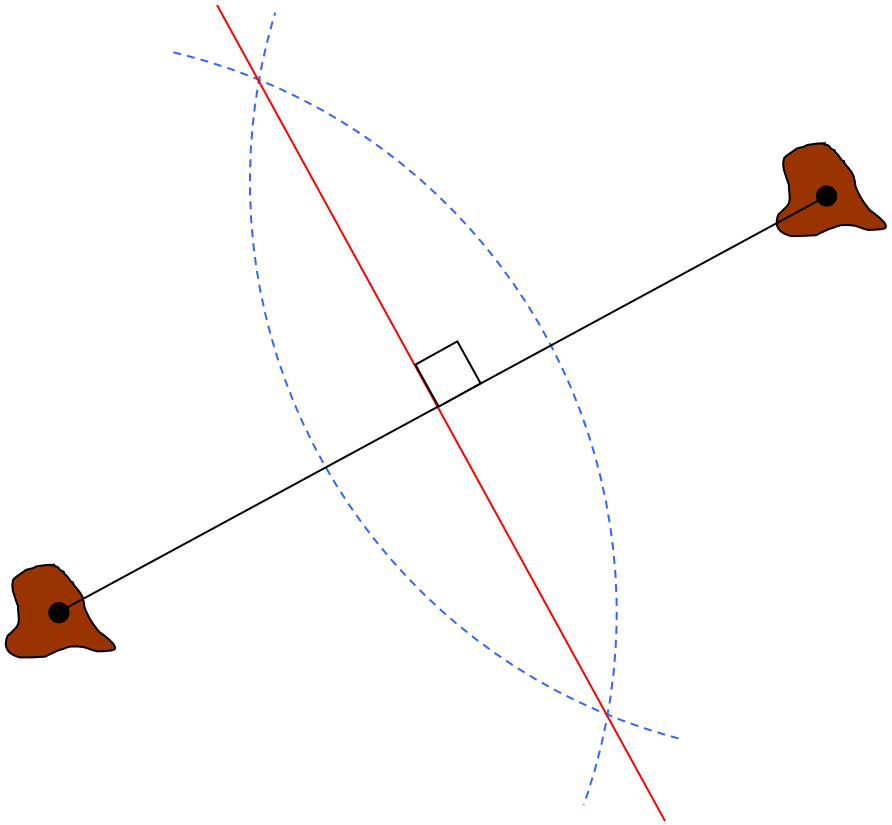
Eg if the compass is set to 4cm every point on the arc will be 4cm from the centre

The **ancient Greeks** were fine Mathematicians, and quite obsessed with arcs.

They enjoyed the challenge of **constructing** different shapes using only a **compass** and a **straight edge** (a ruler with no markings).

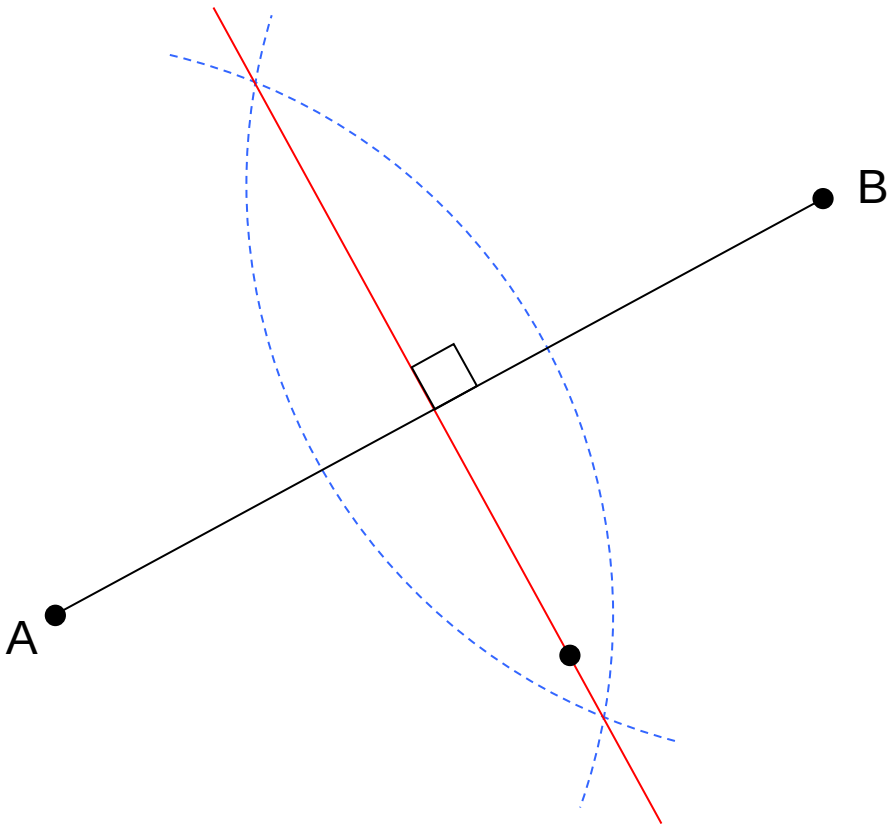
The ship wants to keep as far away from the rocks as possible at all times. What path should the ship take?





This construction is known as the perpendicular bisector of a line between two points

Straight edge and compass constructions



Draw equal arcs, centred on each end of the line, so that they intersect above and below the line

Connect the intersections to make the perpendicular bisector of the line

Any point on the bisector is equidistant from A and B

Perpendicular bisector

The ideal flight path is midway between the maximum and minimum angle of descent. What path should the plane take?

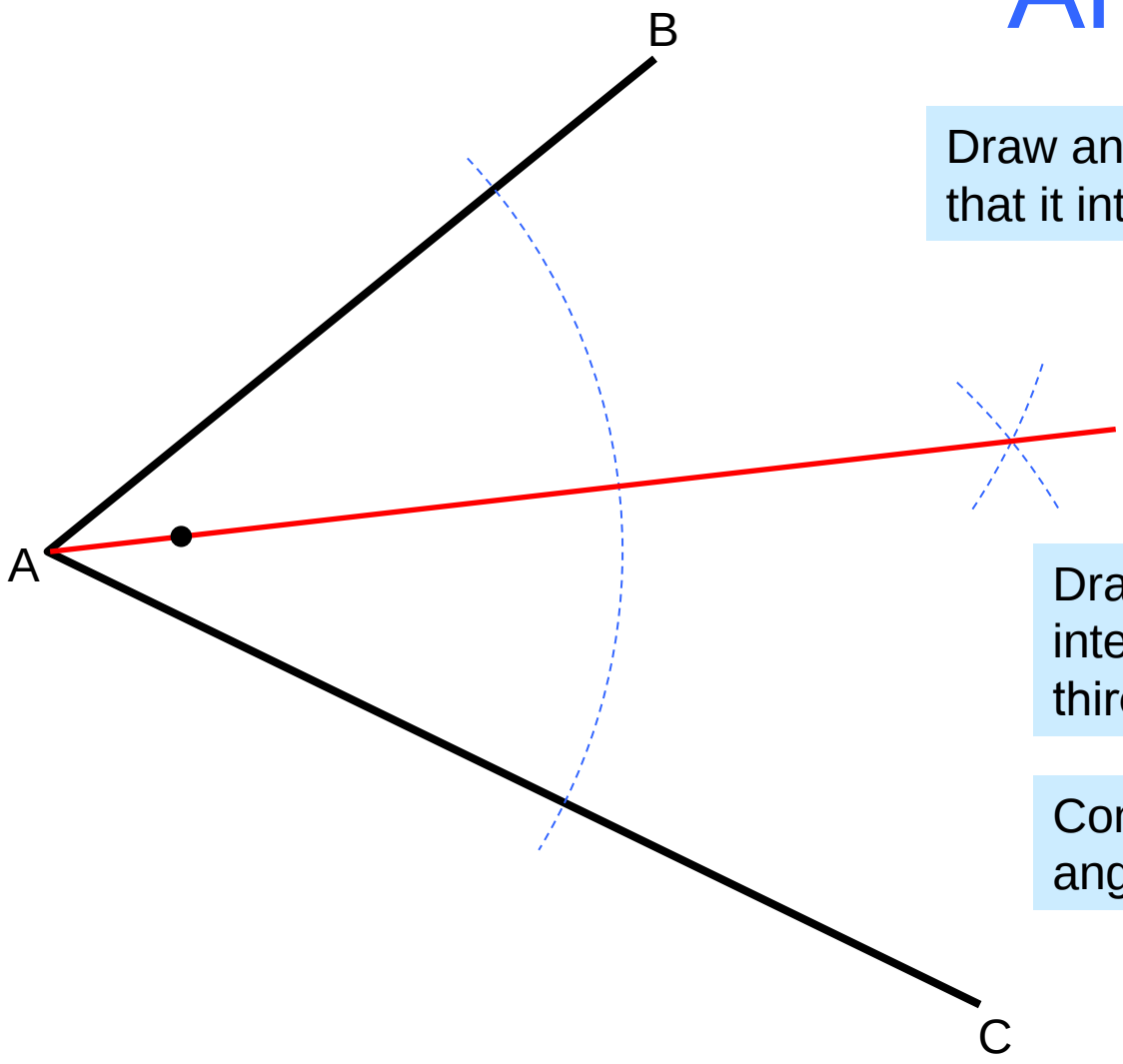
Maximum angle of descent

Minimum angle of descent

This construction is known as the angle bisector between two lines

Straight edge and compass constructions

Angle bisector



Draw an arc, centred on the angle, so that it intersects both sides of the angle

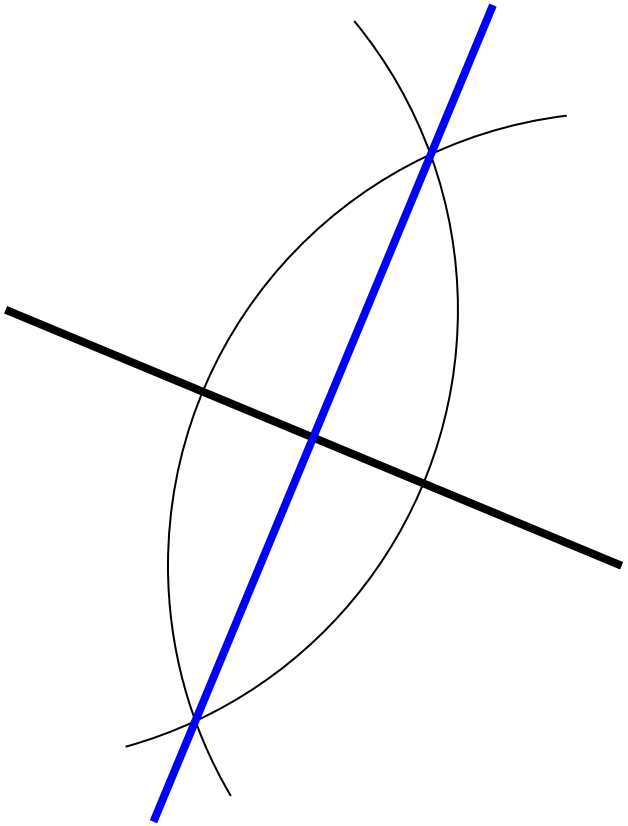
Draw two more arcs, centred on the intersections, so that they make a third intersection ‘within’ the angle

Connect the third intersection to the angle to make the angle bisector

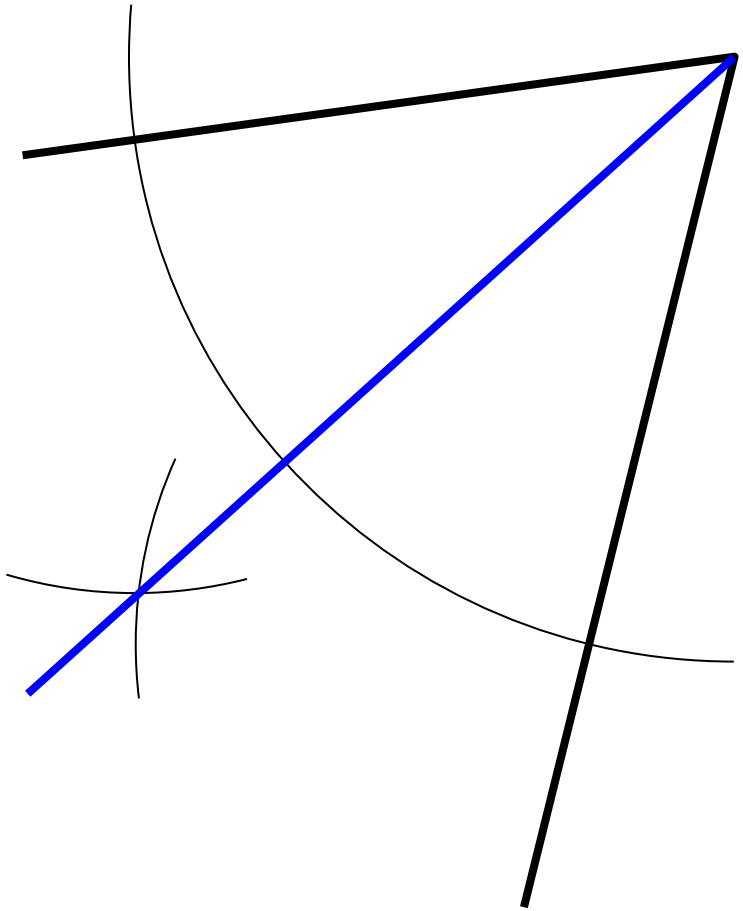
Any point on the bisector is equidistant from AB and AC

Straight edge and compass constructions

1. Construct the perpendicular bisector of this line



2. Use straight edge and compass to bisect this angle



Challenge: perpendicular pandemonia

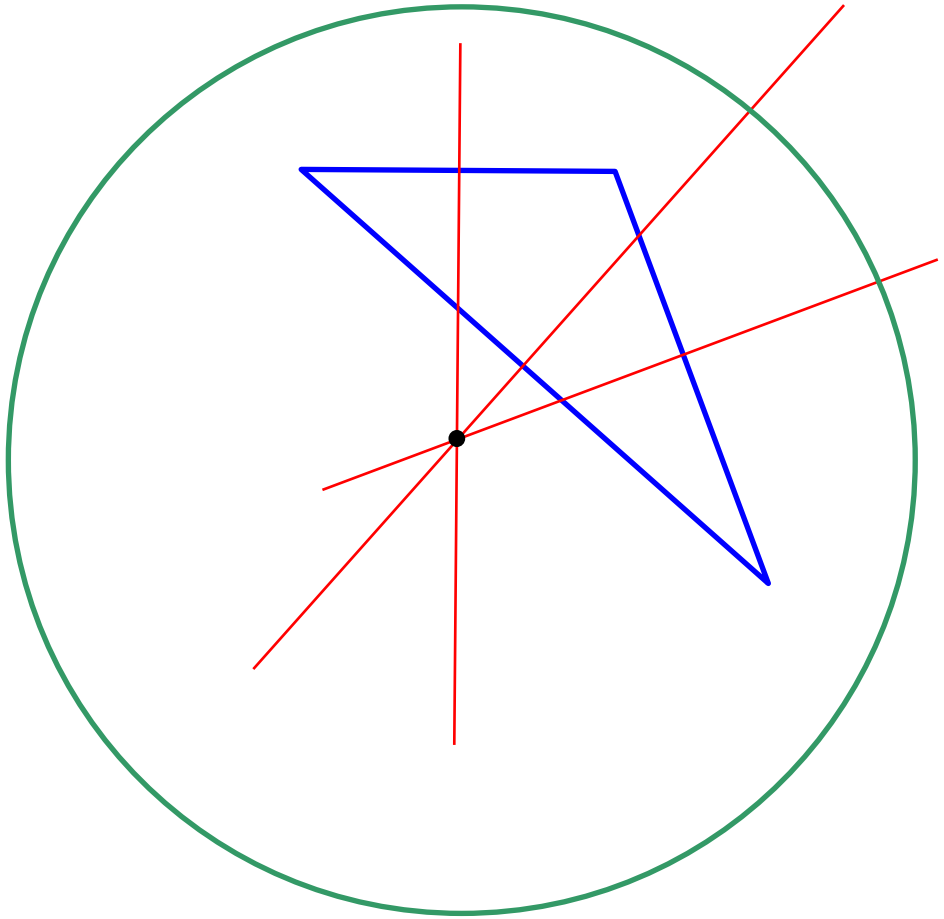
Draw a triangle and construct the perpendicular bisector of each side

You should find that the bisectors intersect at a single point, either inside or outside the circle!

Now using the point of intersection as centre, draw the **smallest** possible circle that does not **enter** the triangle

You should find that the circle touches all three corners of the triangle

This works with **any** triangle…



Challenge: bisector bedlam

Draw a triangle and construct the angle bisector of each corner

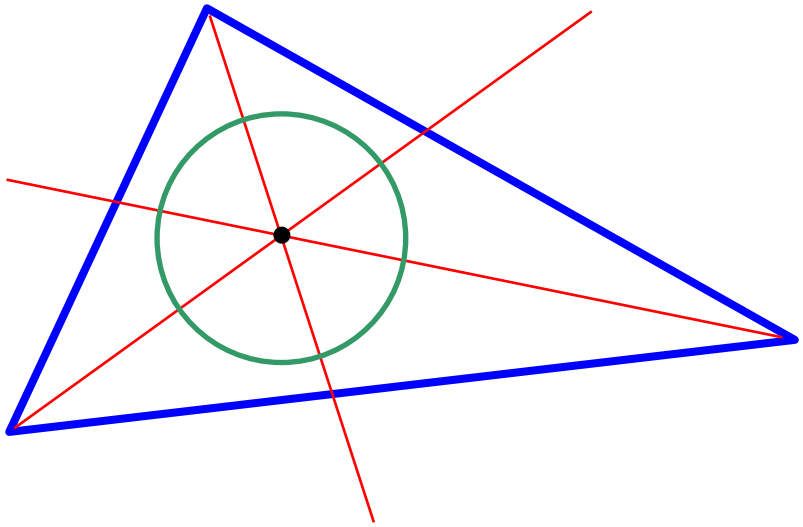
You should find that the bisectors intersect at a single point inside the circle!

Now using the point of intersection as centre, draw the **largest** possible circle that does not **leave** the triangle

You should find that the circle touches all three sides of the triangle

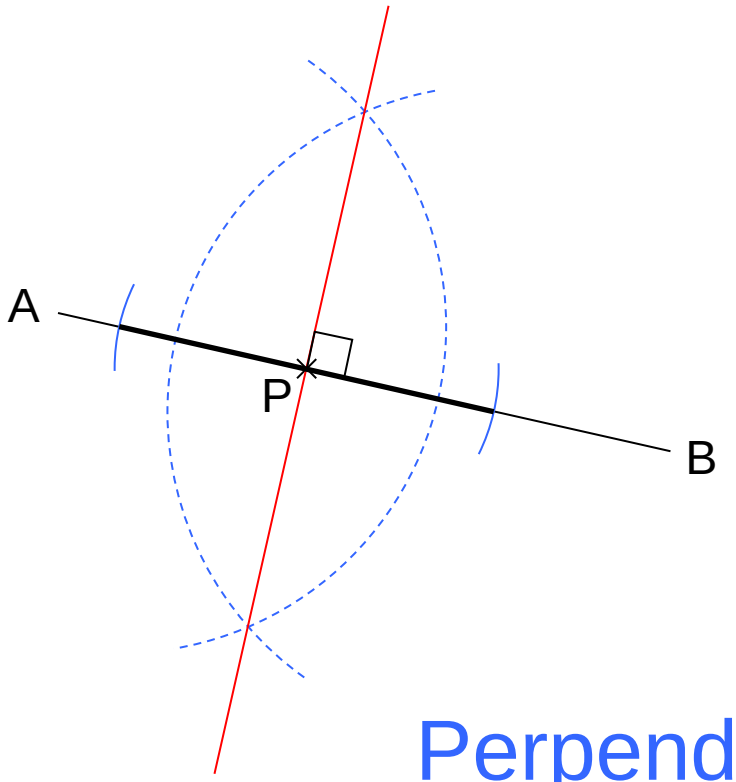
This works with **any** triangle…

Can you explain why this happens?!



Straight edge and compass constructions

You may be asked to construct a line perpendicular to a given line, that also passes through a given point on the line:



Use P as centre to draw arcs and obtain 2 points on the line that are equidistant from P

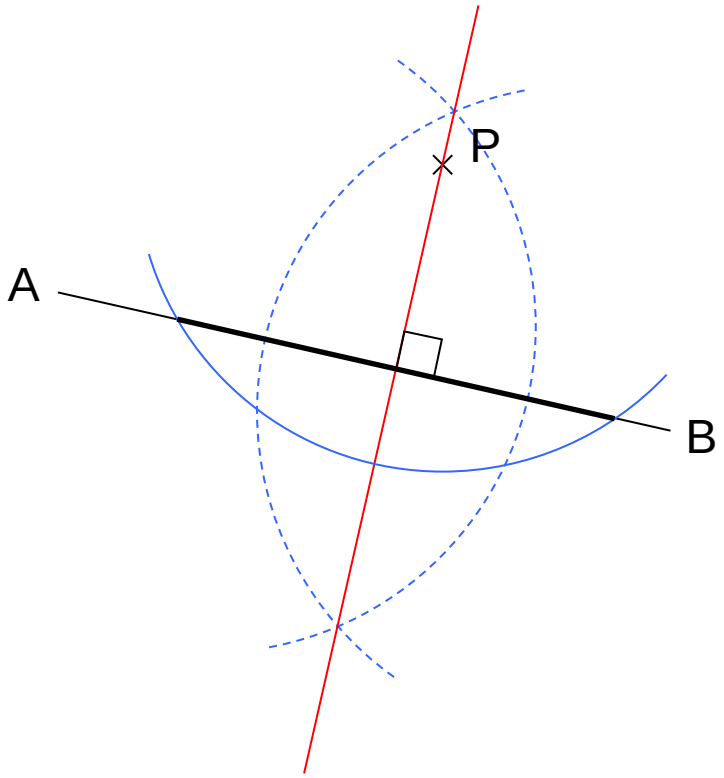
This effectively gives a shorter line with P as its midpoint

Obtain the perpendicular bisector of this shorter line segment- it must pass through the midpoint P

Perpendicular to a point on a line

Straight edge and compass constructions

You may be asked to construct a line perpendicular to a given line, that also passes through a given point **not** on the line:



Use P as centre to draw an arc and obtain 2 points on the line that are equidistant from P

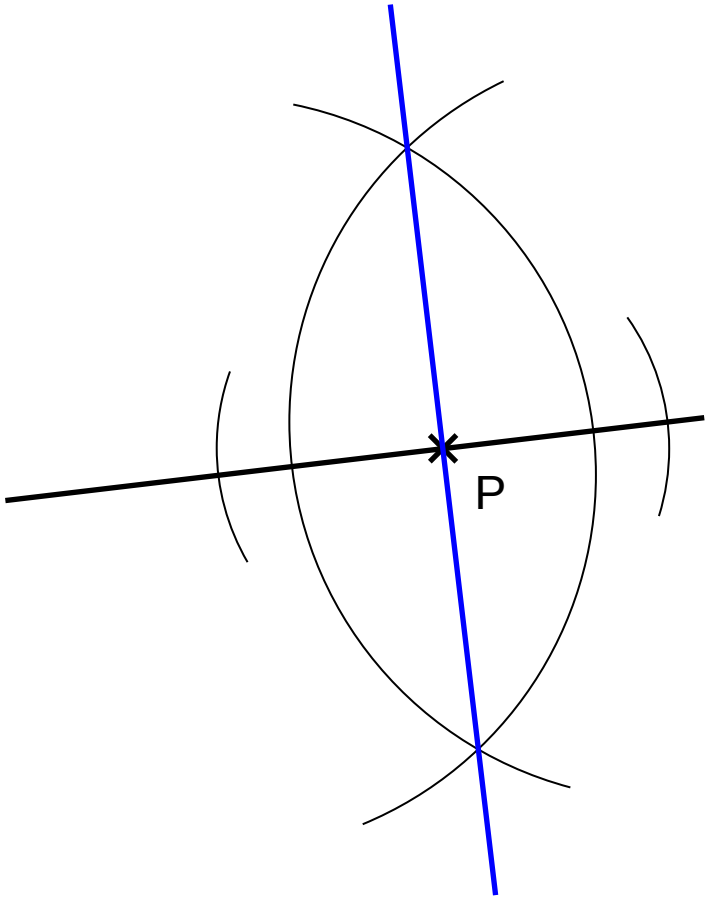
This effectively gives a shorter line with P as its midpoint

Obtain the perpendicular bisector of this shorter line segment- it must pass through the midpoint P

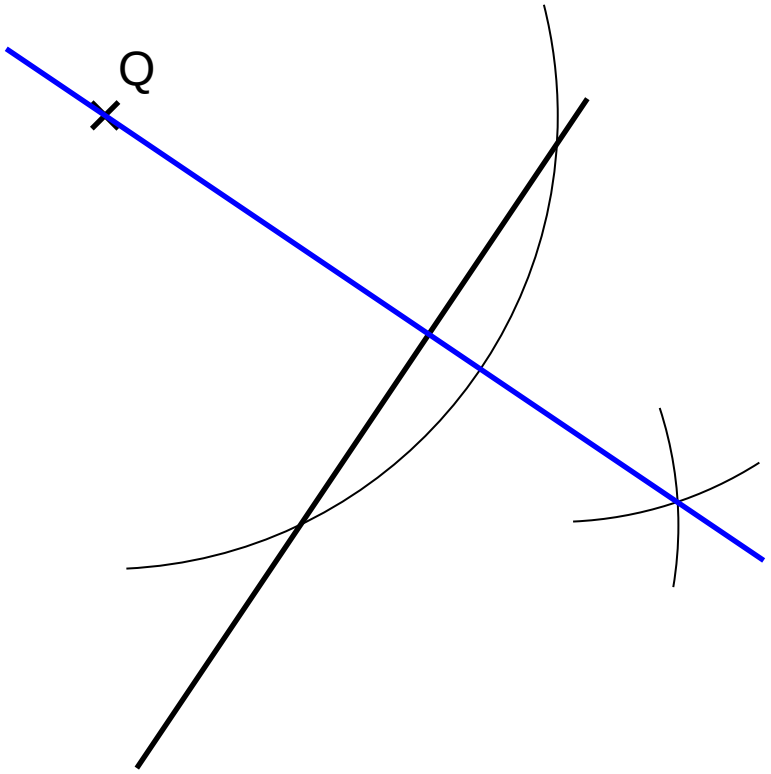
Perpendicular from a point to a line

Straight edge and compass constructions

3. Construct a perpendicular to the given line from the point P

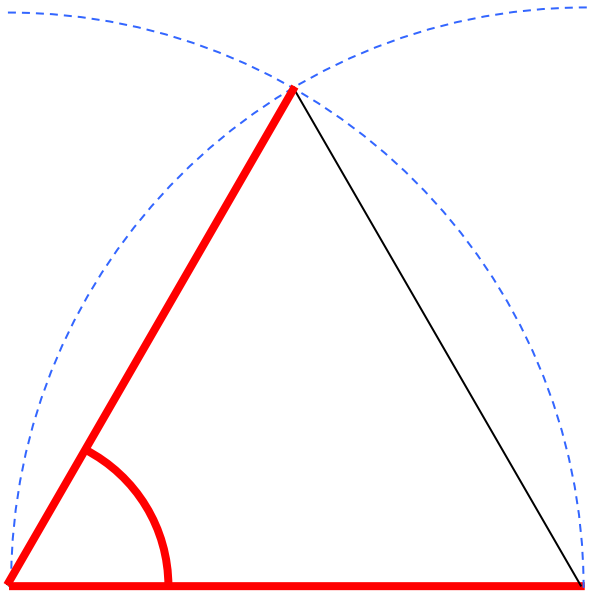


4. Construct a perpendicular from the given line to the point Q



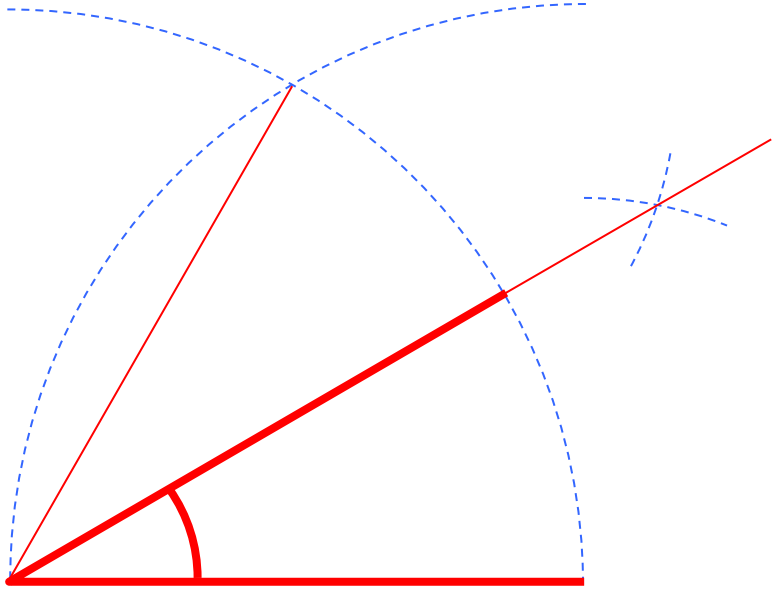
Angle constructions (compass only)

60^o angle



Use the equilateral triangle construction

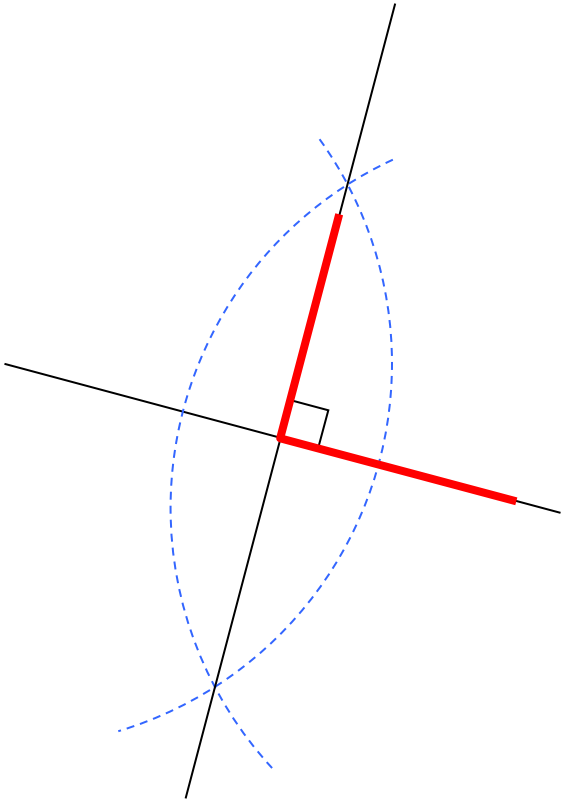
30^o angle



Bisect a 60^o angle from an equilateral triangle

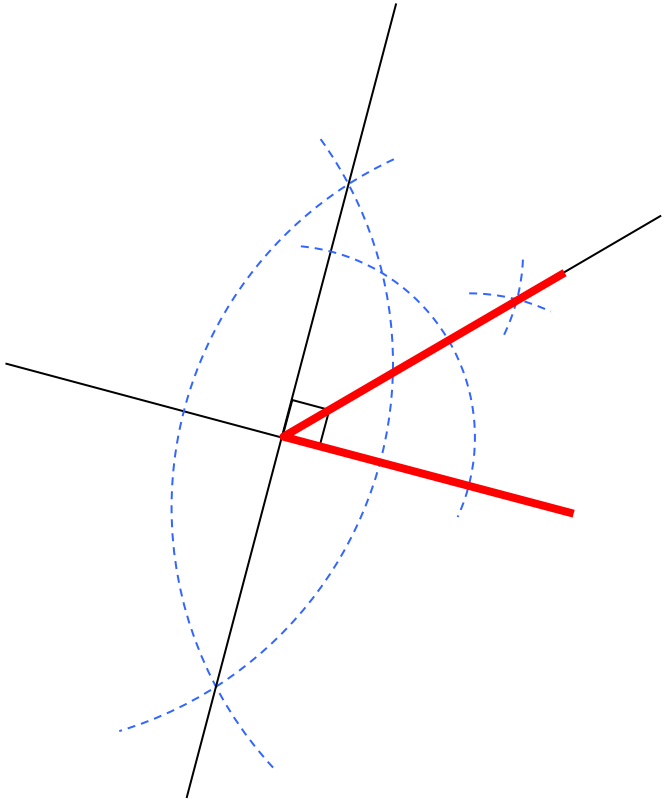
Angle constructions (compass only)

Right-angle



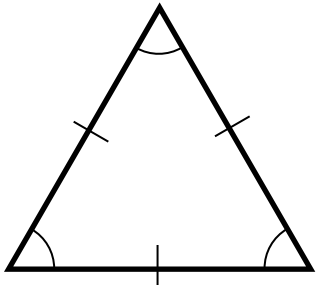
Draw a perpendicular bisector

45^{o} angle

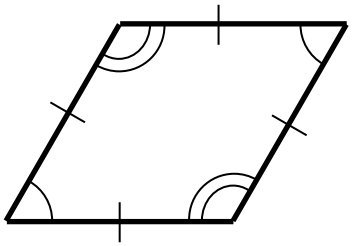


Bisect a 90^{o} angle from
a perpendicular bisector

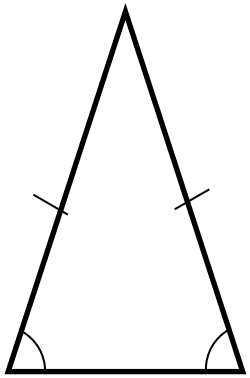
Shape properties



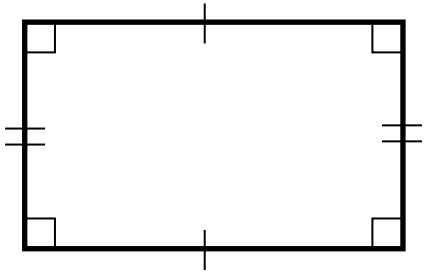
Equilateral triangle



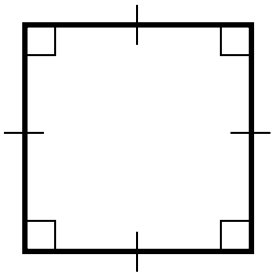
Rhombus



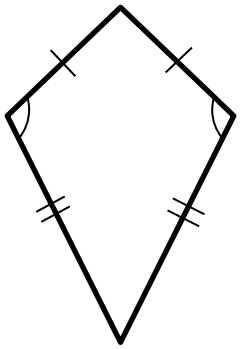
Isosceles triangle



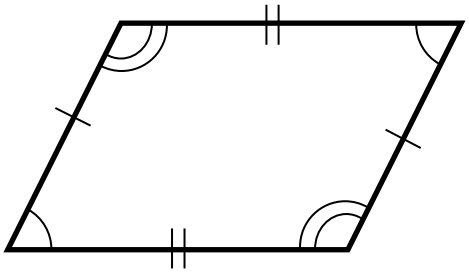
Rectangle



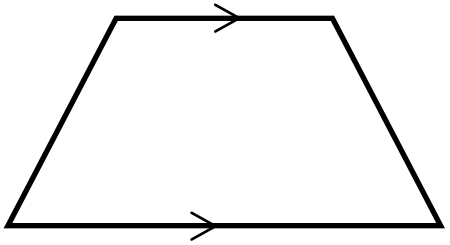
Square



Kite



Parallelogram

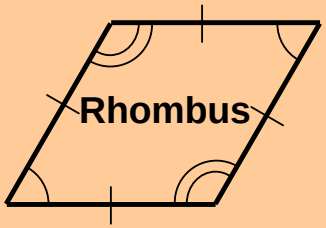


Trapezium

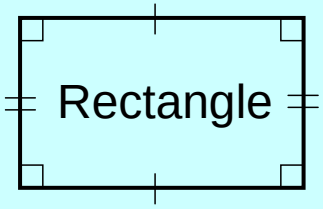
What shape am I?

Name the shapes, based on the description

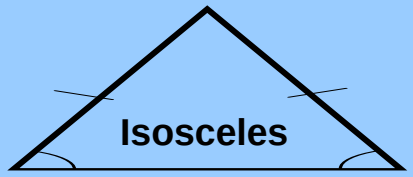
I am a quadrilateral with 4 equal sides and no right angles



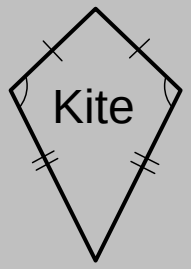
I am a quadrilateral with 2 pairs of equal sides and 4 right angles



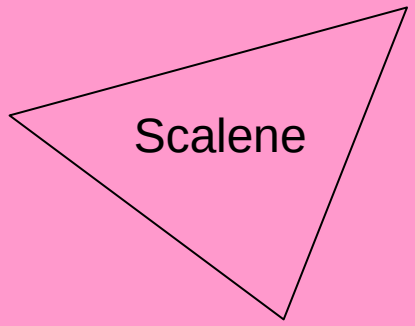
I am a triangle with 2 equal sides and 2 equal angles



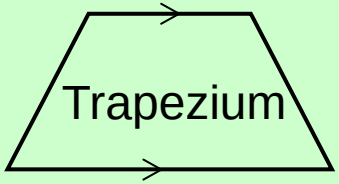
I am a quadrilateral with 2 pairs of adjacent, equal sides

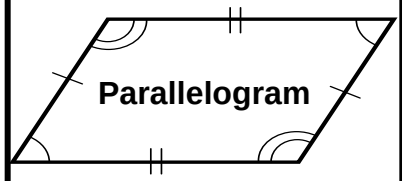
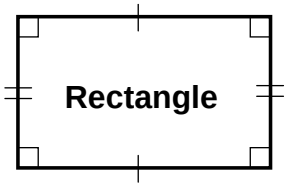
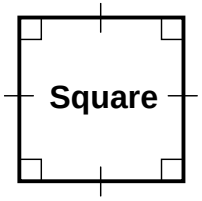
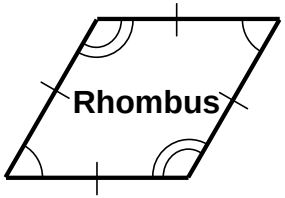
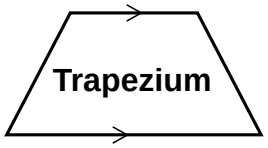


I am a triangle with all sides different



I am a quadrilateral with 1 pair of parallel sides



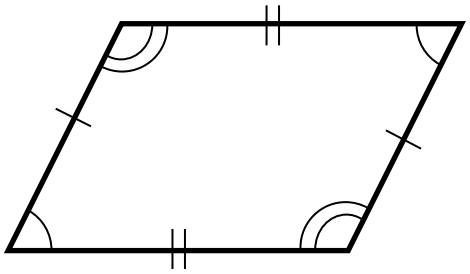
	Sides	Angles
		
		
		
		
		
		

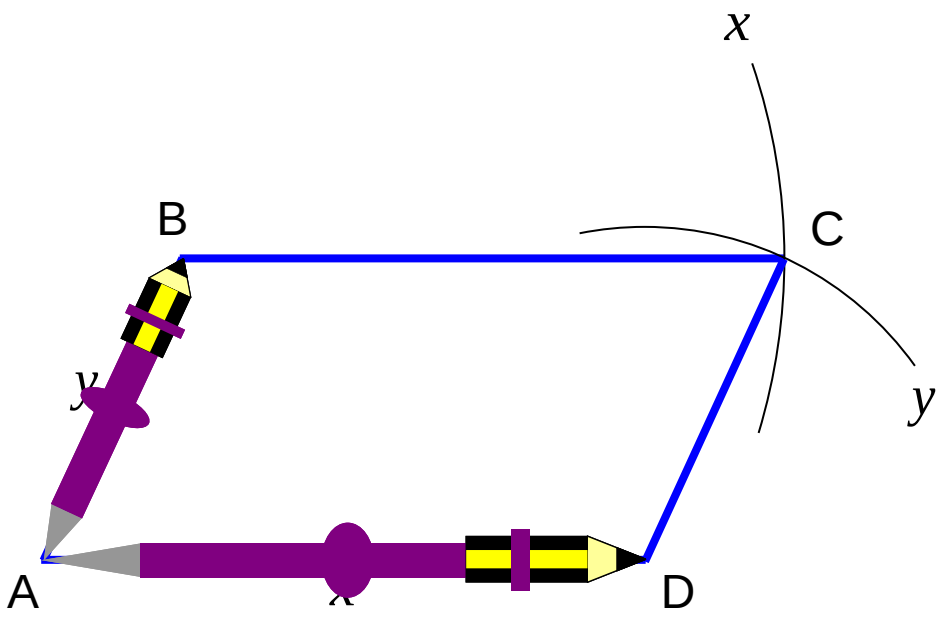
Match up the statements with the correct shapes

Properties of sides	Properties of angles
Two pairs of adjacent sides are equal but not all 4 sides are equal	Only one pair of opposite angles are equal
Both pairs of opposite sides are equal & parallel	Both pairs of opposite angles are equal
All 4 sides equal. Opposite sides are parallel	Both pairs of opposite angles are equal
One pair of opposite sides are parallel	All 4 angles are right angles
All 4 sides equal. Opposite sides are parallel	No specific angles properties
Both pairs of opposite sides are equal & parallel	All 4 angles are right angles

We can use shape properties to **construct** shapes using a straight edge and compasses

Eg Parallelogram





Draw 2 lines to any lengths x and y

C must be x away from B , so draw an arc of radius x, centred on B

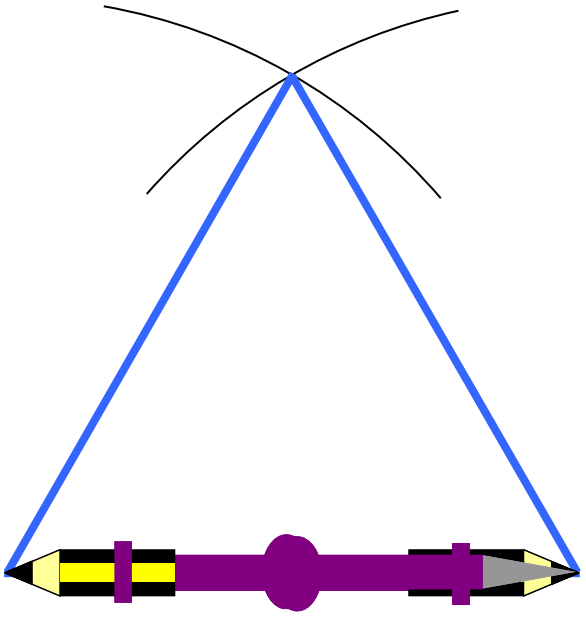
C must also be y away from D , so draw an arc of radius y, centred on D

C must be at the intersection of these arcs

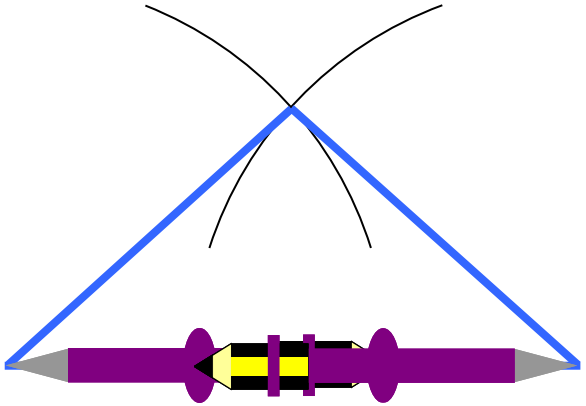
Straight edge and compass constructions

Use a straight edge and compass to accurately construct:

1. Equilateral triangle



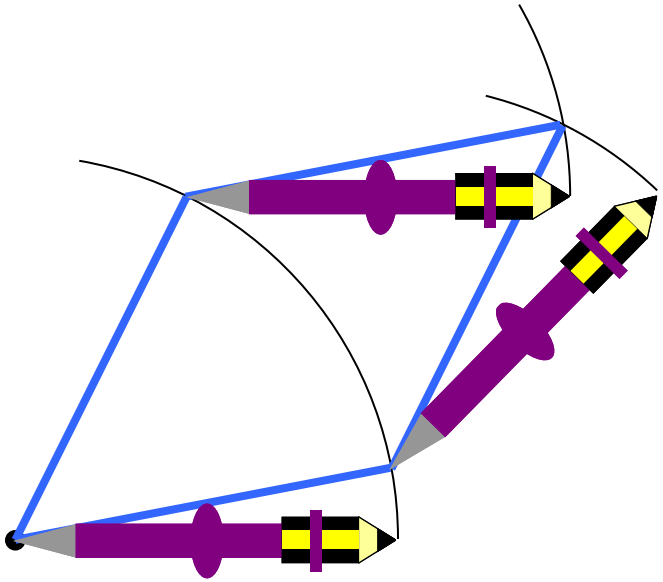
2. Isosceles triangle



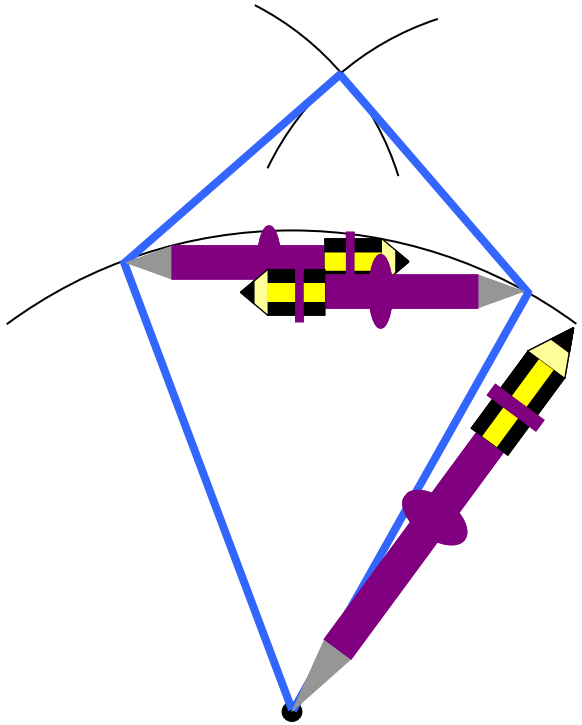
Straight edge and compass constructions

Use a straight edge and compass to accurately construct:

3. Rhombus



4. Kite

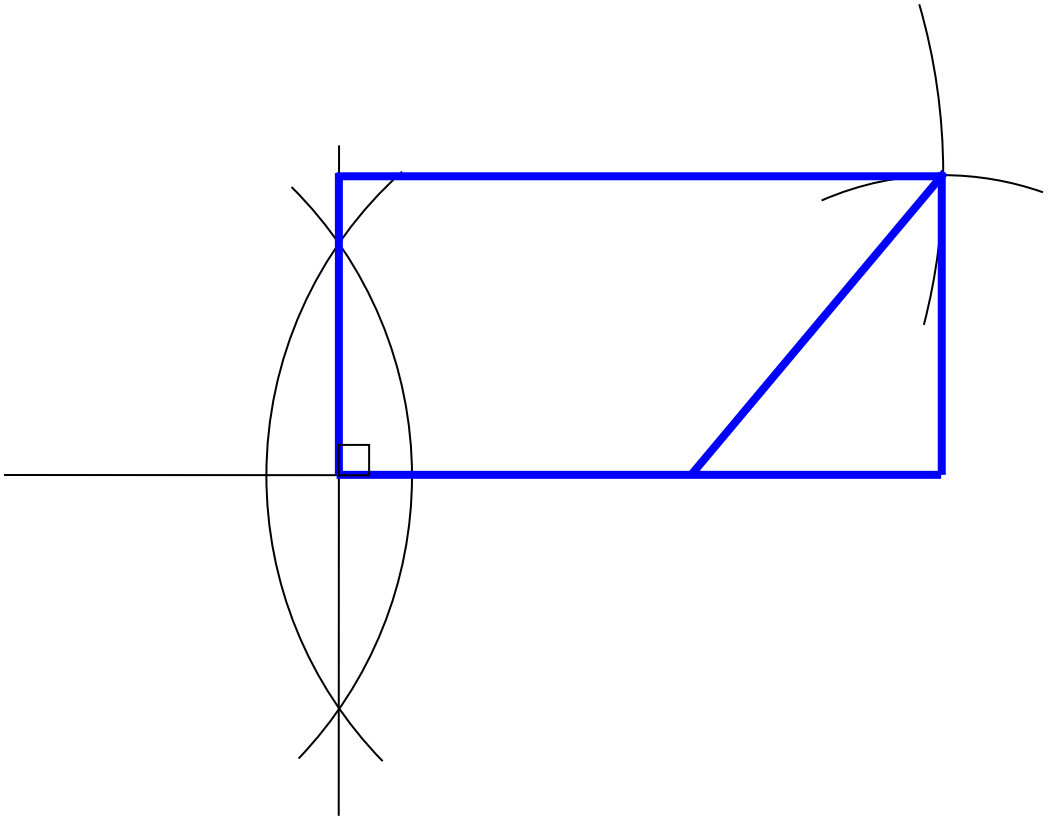


Straight edge and compass constructions

Use a straight edge and compass to accurately construct:

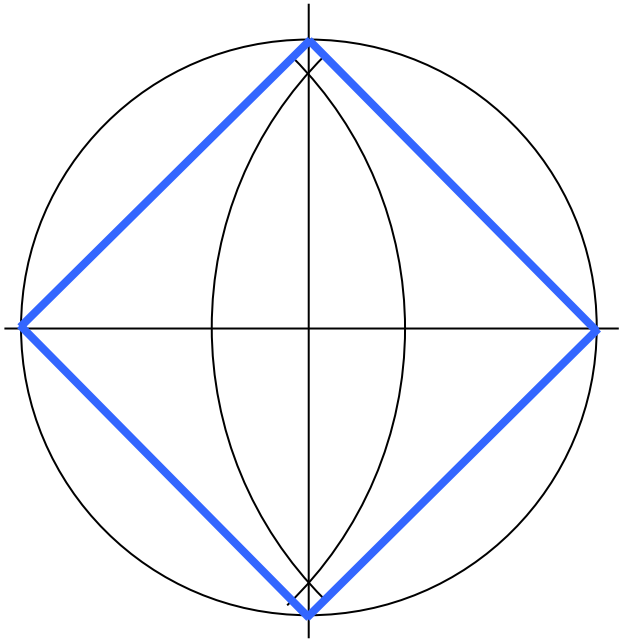
5. Rectangle

6. Trapezium

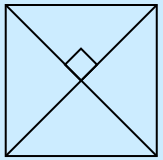


Straight edge and compass constructions

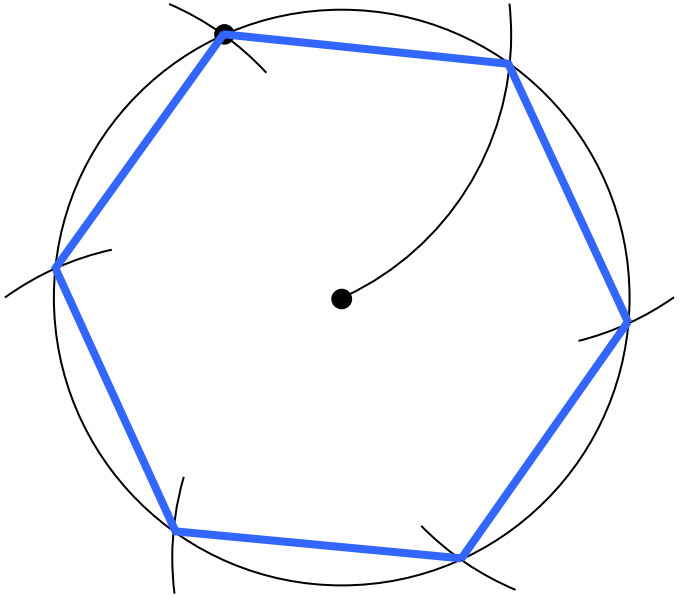
7. Square



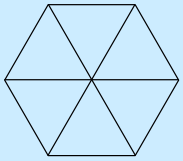
Hint: Use property that the diagonals of a square bisect each other at right-angles



8. Regular hexagon



Hint: Use property that a regular hexagon consists of 6 equilateral triangles



Any regular polygon can be drawn inside a circle, provided you can work out the length of the sides of the polygon. This is harder than you might think!

Think those are hard?

Mathematicians have investigated which regular polygons can be constructed for over 2000 years.

They now know that some polygons can be constructed (eg 3,4,5,6,8,10,12,15,… sided) but some cannot (eg 7,9,11,1314,18,…sided).

In 1796 the famous Mathematician Gauss proved, using algebra, that the number of sides had to fit a special pattern in order to be constructible.

However it took other Mathematicians years to work out how to actually draw some of these:

5 sided (pentagon)

17 sided (heptadecagon) by Erchinger in about 1800

257-gon by Richelot in 1832

65537-gon by Hermes in 1894 (taking 10 years to do so!)

Constructing a regular pentagon

1. Draw circle, marking centre O

2. Draw any diameter AB

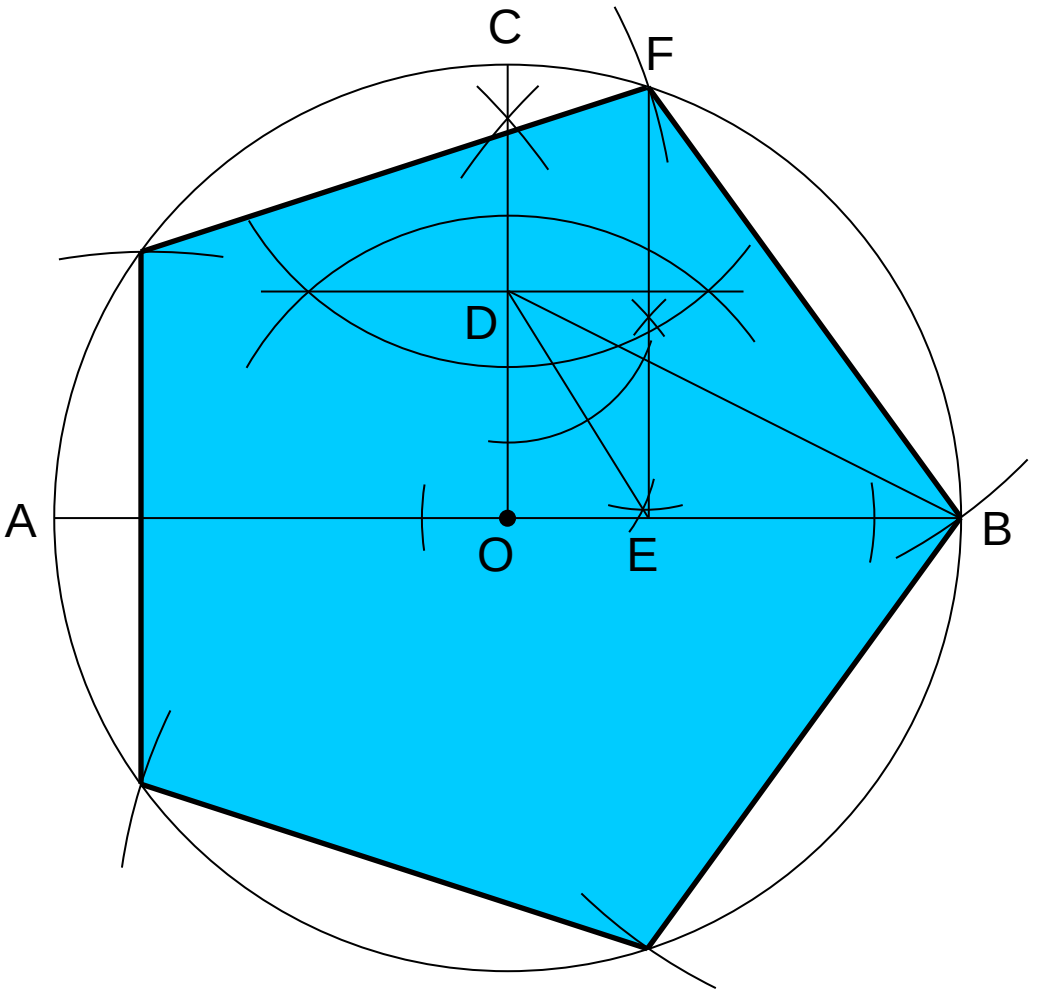
3. Construct radius perpendicular to AB, to obtain C on circumference

4. Find midpoint D of OC and connect DB

5. Bisect angle ODB to obtain E on OB

6. Construct perpendicular to OB from E to obtain F on circumference

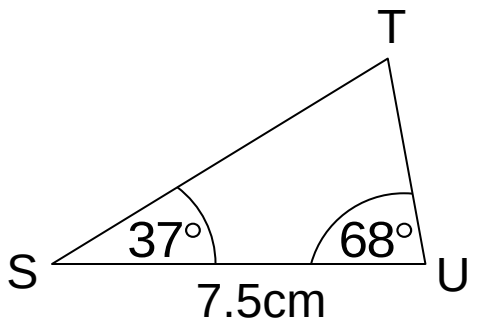
7. Set compass to radius BF and centre F, repeatedly mark circumference till you return to F (5 points), then connect adjacent points.



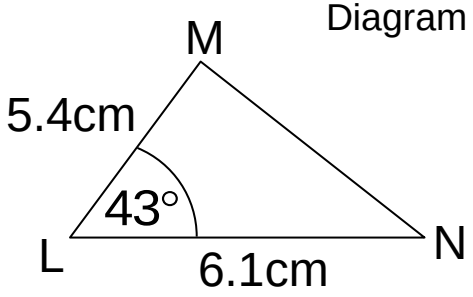
Ruler and protractor constructions

Use a ruler and protractor to **construct** each of the triangles shown.
You must show all construction lines.

1. Diagram NOT accurate



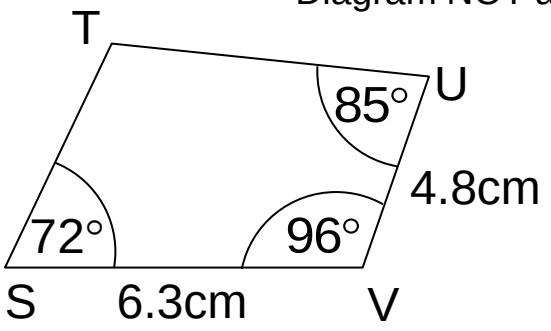
2. Diagram NOT accurate



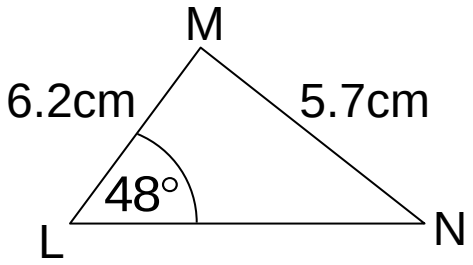
Ruler and protractor constructions

Use a ruler and protractor to **construct** each of the shapes shown.
You must show all construction lines.

3. Diagram NOT accurate



4. Diagram NOT accurate



Can you show **both** answers?

Ruler and compass constructions

Use a ruler and compasses to **construct** each of the triangles shown.
You must show all construction lines.

1.

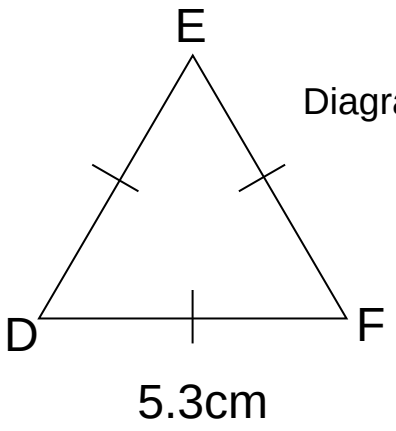
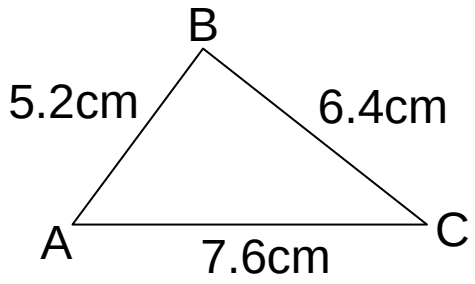


Diagram NOT accurate

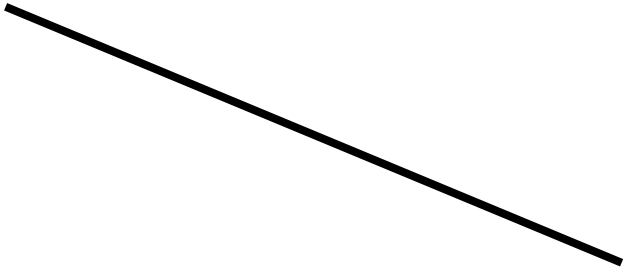
2.

Diagram NOT accurate

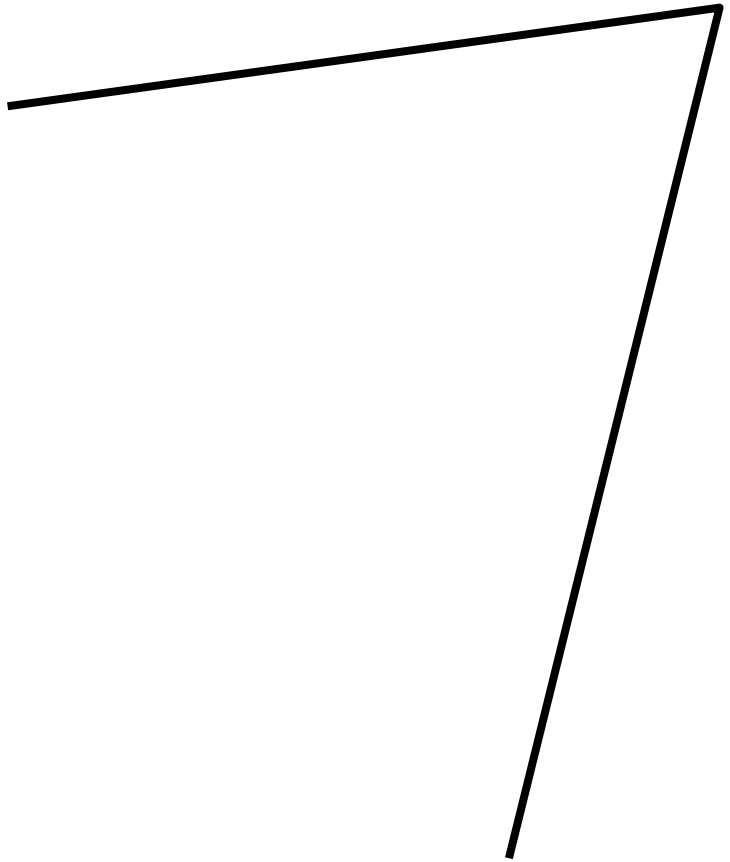


Straight edge and compass constructions

1. Construct the perpendicular bisector of this line



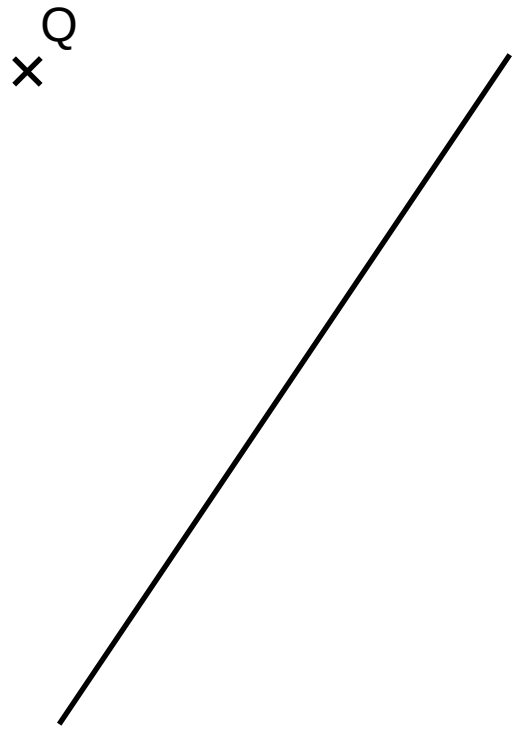
2. Use straight edge and compass to bisect this angle



Straight edge and compass constructions

3. Construct a perpendicular to the given line from the point P

4. Construct a perpendicular from the given line to the point Q



Angle constructions (compass only)

60^o angle

30^o angle

Angle constructions (compass only)

Right-angle

45^{o} angle

Straight edge and compass constructions

Use a straight edge and compass to accurately construct:

1. Equilateral triangle

2. Isosceles triangle

Straight edge and compass constructions

Use a straight edge and compass to accurately construct:

3. Rhombus

4. Kite

Straight edge and compass constructions

Use a straight edge and compass to accurately construct:

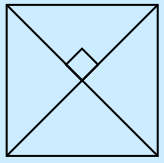
5. Rectangle

6. Trapezium

Straight edge and compass constructions

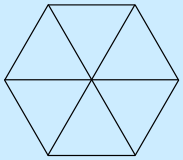
7. Square

Hint: Use property that the diagonals of a square bisect each other at right-angles



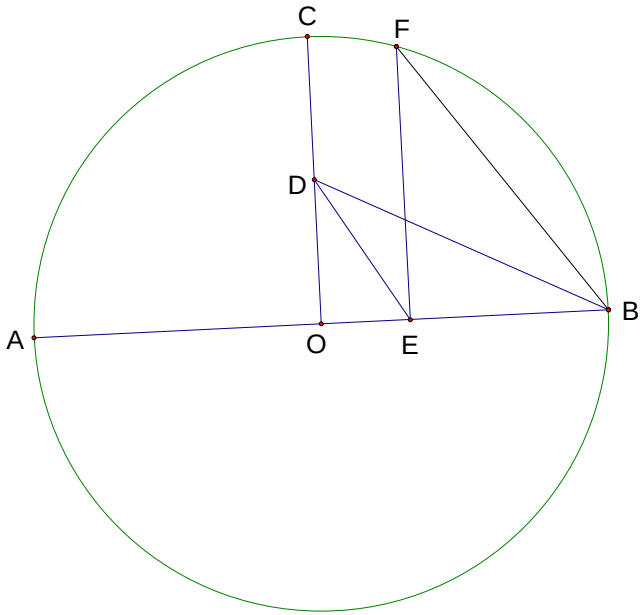
8. Regular hexagon

Hint: Use property that a regular hexagon consists of 6 equilateral triangles



Challenge: can you follow these instructions to accurately construct a regular pentagon using only straight edge & compasses?

1. Draw circle, marking centre O
2. Draw any diameter AB
3. Construct radius perpendicular to AB, to obtain C on circumference
4. Find midpoint D of OC and connect DB
5. Bisect angle ODB to obtain E on OB
6. Construct perpendicular to OB from E to obtain F on circumference
7. Set compass to radius BF and centre F, repeatedly mark circumference till you return to F (5 points), then connect adjacent points.



Leads to