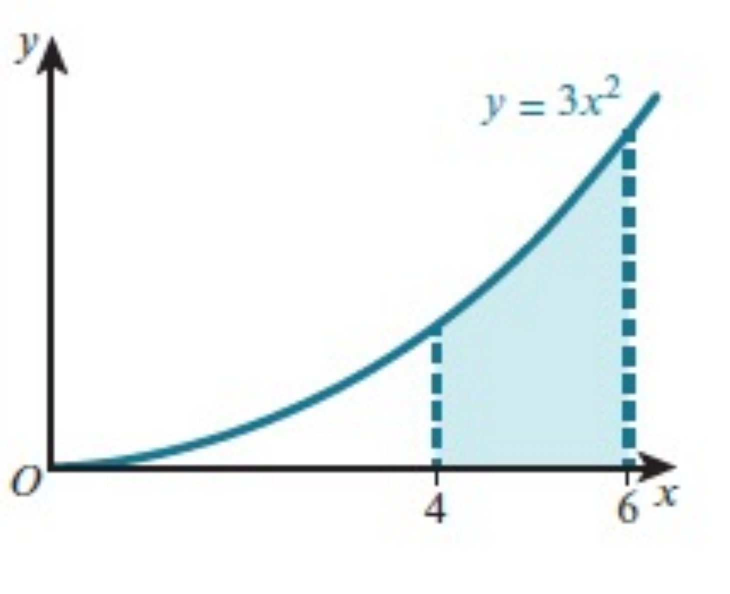
AREA UNDER A CURVE

As Level

If $y = f(x)$ is a function with $y \geq 0$, then area, A, bounded by the curve $y = f(x)$, the x-axis and the lines $x = a$ and $x = b$ is given by the formula $A = \int_a^b y dx$.

Example 1: Find the area of the shaded region.



$$\int_4^6 3x^2 dx =$$

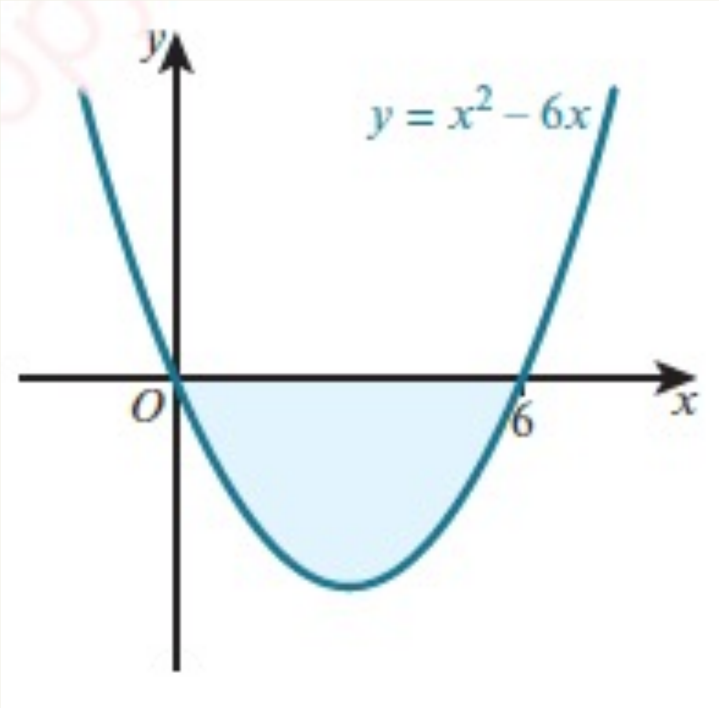
$$[x^3]_4^6 =$$

$$6^3 - 4^3 = 216 - 64 =$$

$$= 152$$

The required area is above the $x - axis$.

Example 2: Find the area of the shaded region.



$$\int_0^6 (x^2 - 6x)dx =$$

$$= [\frac{1}{3}x^3 - 3x^2]_0^6 =$$

$$= \frac{1}{3}(6^3 - 0^3) - 3(6^2 - 0^2) =$$

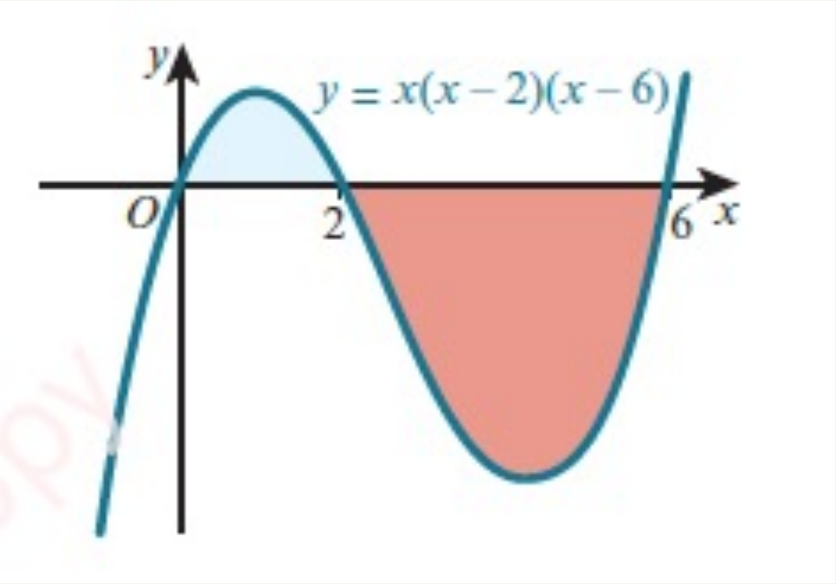
$$= \frac{216}{3} - 108 = 72 - 108$$

$$= -36$$

The required area lies below the $x - axis$, then will have a negative value.

Area is 36

Example 3: Find the total area of the shaded regions.



$$\int_0^2 x(x-2)(x-6)dx =$$

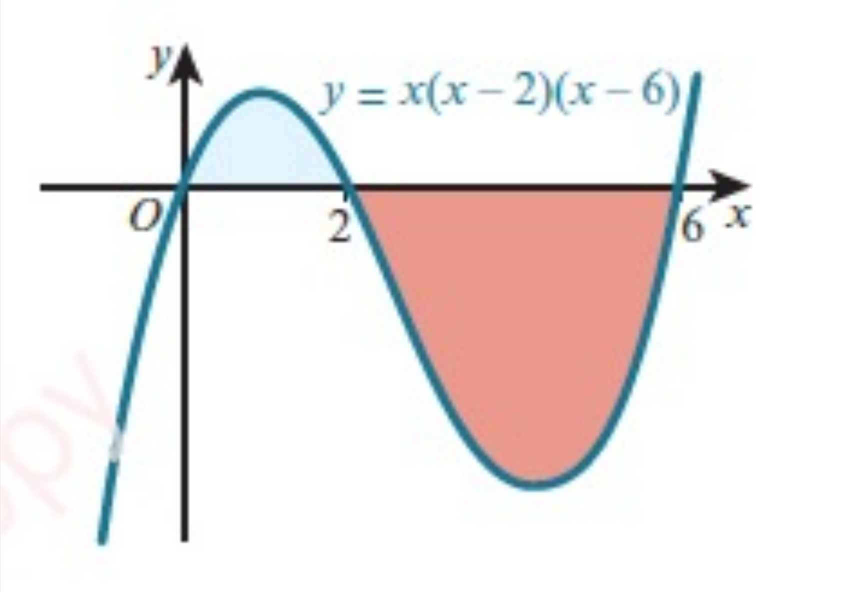
$$\int_0^2 (x^3 - 8x^2 + 12x)dx =$$

$$= [\frac{1}{4}x^4 - \frac{8}{3}x^3 + 6x^2]_0^2 =$$

$$= \frac{1}{4}(2^4 - 0^4) - \frac{8}{3}(2^3 - 0^3) + 6(2^2 - 0^2)$$

$$= 6\frac{2}{3}$$

Example 3: Find the total area of the shaded regions.



$$\int_2^6 x(x-2)(x-6)dx =$$

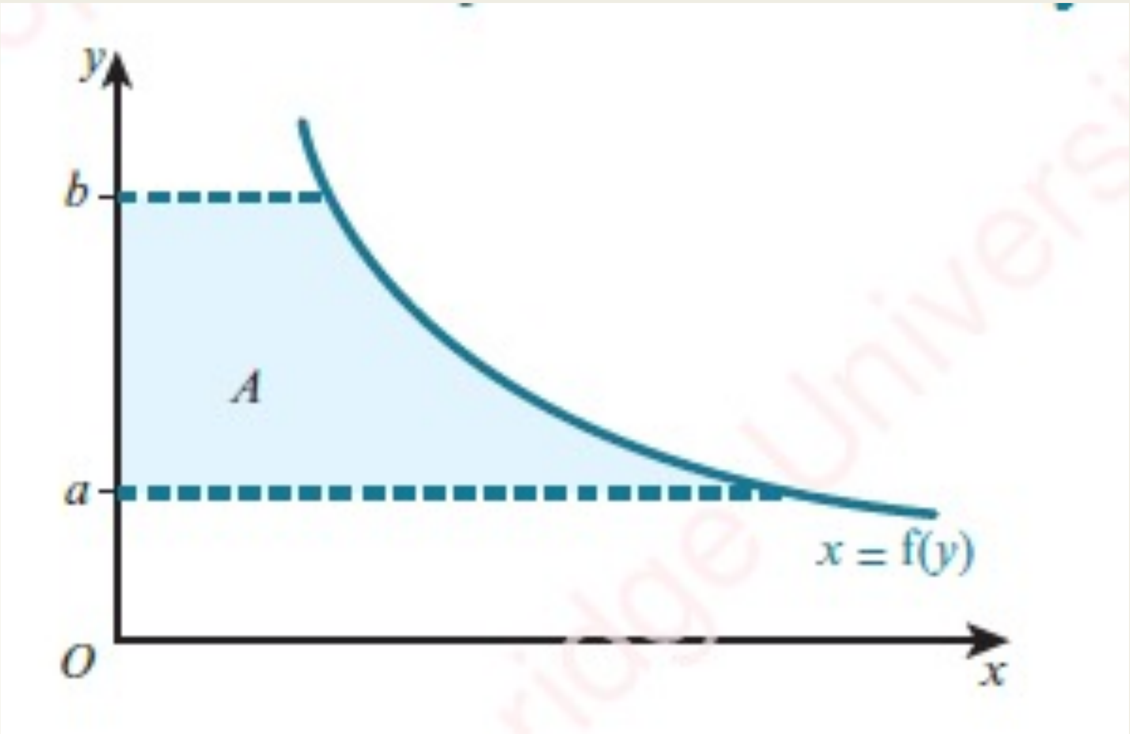
$$\int_2^6 (x^3 - 8x^2 + 12x)dx =$$

$$= [\frac{1}{4}x^4 - \frac{8}{3}x^3 + 6x^2]_2^6 =$$

$$= \frac{1}{4}(6^4 - 2^4) - \frac{8}{3}(6^3 - 2^3) + 6(6^2 - 2^2) \quad = -42\frac{2}{3}$$

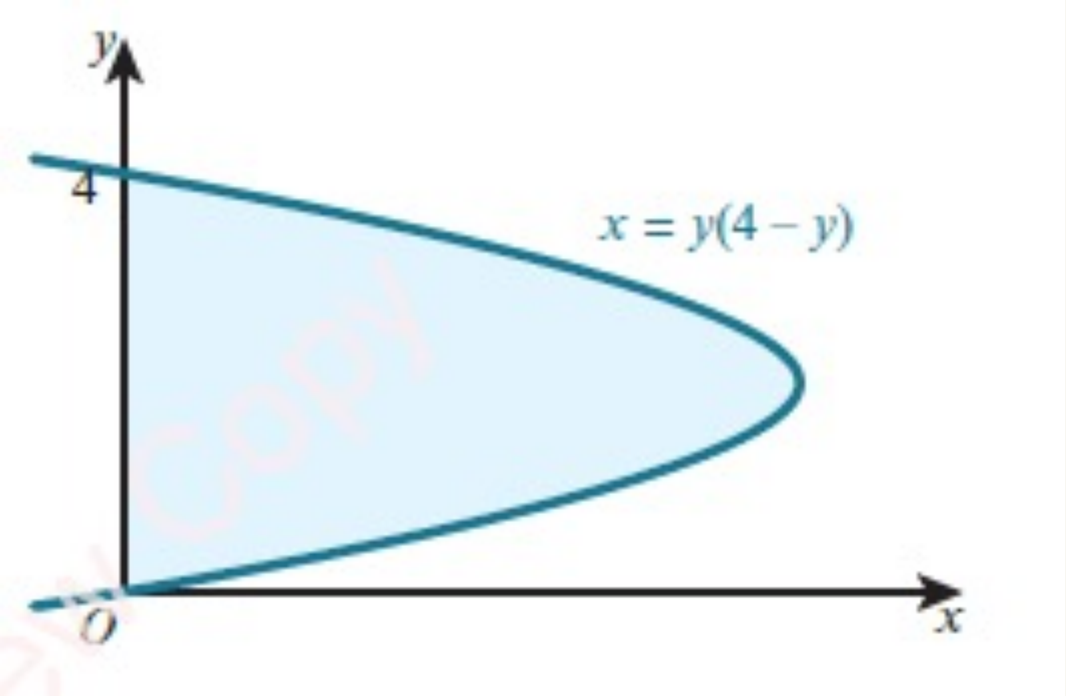
$$= 6\frac{2}{3} + 42\frac{2}{3} = 49\frac{1}{3}$$

Area enclosed by a curve and the y-axis



If $x = f(y)$ is a function with $x \geq 0$, then area, A, bounded by the curve $x = f(y)$, the y-axis and the lines $y = a$ and $y = b$ is given by the formula $A = \int_a^b x dy$ when $x \geq 0$.

Example 4: Find the area of the shaded region.



$$\int_0^4 (4y - y^2)dy =$$

$$= [2y^2 - \frac{1}{3}y^3]_0^4 =$$

$$= 2(4^2 - 0^2) - \frac{1}{3}(4^3 - 0^3)$$

$$= 10\frac{2}{3}$$