Elastic collisions

Key terms

Term (symbol)	Meaning
Closed system	System that is not acted on by a net external force. Also called an isolated system.
Conservation of momentum	In a closed (isolated) system, momentum is constant.
Elastic collision	Collision where both momentum and kinetic energy are conserved. There is no change in kinetic energy in the system as a result of the collision.

Equations

Equation	Symbols	Meaning in words
$p_i = p_f$	p_i and p_f are the total initial and final momentum	The total initial momentum equals the total final momentum for a closed system. Commonly called the conservation of momentum.

For two objects with mass m_1 and m_2, the conservation of momentum equation can be written:

$$p_i = p_f$$

$$p_{1i} + p_{2i} = p_{1f} + p_{2f}$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

Where:

- p_{1i} is the initial momentum of the first object
- p_{2i} is the initial momentum of the second object
- p_{1f} is the final momentum of the first object
- p_{2f} is the final momentum of the second object
- v_{1i} is the initial velocity of the first object
- v_{2i} is the initial velocity of the second object
- v_{1f} is the final velocity of the first object
- v_{2f} is the final velocity of the second object

How to predict final velocities for an elastic collision

We know a collision is elastic if kinetic energy is conserved:

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

and momentum is conserved:

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

If we imagine ourselves sitting on object 1 moving at velocity v_1, object 2 will look like it is moving at speed $v_1 - v_2$. The difference in the velocities of the two objects tells us how fast object 1 is moving relative to object 2, and is sometimes called the **relative velocity**. If kinetic energy and momentum are conserved, we can make some predictions about the relative velocity before and after the collision.

- The magnitude of the relative velocity is the same before and after the collision. That means if we are sitting on object 1 moving at velocity v_1, object 2 will look like it is moving at the same speed both before and after the collision.

$$|v_{1i} - v_{2i}| = |v_{1f} - v_{2f}|$$

- The relative velocity has opposite signs before and after the collision. If we are sitting on object 1 moving at v_1, object 2 will look like it changes direction after the collision.

$$v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$$

The relative velocity will have these properties before and after an elastic collision for any combination of masses.

Common mistakes and misconceptions

Sometimes people forget that momentum is always conserved, but only in an isolated system. If there is a net external force on the system (an external impulse), then momentum is added to the system, and momentum is not conserved.