# WHAT IS CONSERVATION OF MOMENTUM?

## WHAT IS THE PRINCIPLE OF CONSERVATION OF MOMENTUM?

In physics, the term *conservation* refers to something which doesn't change. This means that the variable in an equation which represents a conserved quantity is constant over time. It has the same value both before and after an event.

There are many conserved quantities in physics. They are often remarkably useful for making predictions in what would otherwise be very complicated situations. In mechanics, there are three fundamental quantities which are conserved. These are *momentum*, *energy*, and *angular momentum*. Conservation of momentum is mostly used for describing collisions between objects.

Just as with the other conservation principles, there is a catch: conservation of momentum applies only to an *isolated system* of objects. In this case an isolated system is one that is not acted on by force external to the system—i.e., there is no *external impulse*. What this means in the practical example of a collision between two objects is that we need to include both objects and anything else that applies a force to any of the objects for any length of time in the system.

If the subscripts *i* and *f* denote the initial and final momenta of objects in a system, then the principle of conservation of momentum says

$$\mathbf{p}_{1i} + \mathbf{p}_{2i} + \ldots = \mathbf{p}_{1f} + \mathbf{p}_{2f} + \ldots$$

## WHY IS MOMENTUM CONSERVED?

Conservation of momentum is actually a direct consequence of Newton's third law. Consider a collision between two objects, object A and object B. When the two objects collide, there is a force on A due to B $F_{AB}$ but because of Newton's third law, there is an equal force in the opposite direction, on B due to A $F_{BA}$.

$$F_{\mathrm{AB}} = -F_{\mathrm{BA}}$$

The forces act between the objects when they are in contact. The length of time for which the objects are in contact $t_{AB}$ and $t_{BA}$ depends on the specifics of the situation. For example, it would be longer for two squishy balls than for two billiard balls. However, the time must be equal for both balls.

$$t_{\mathrm{AB}} = t_{\mathrm{BA}}$$

Consequently, the impulse experienced by objects A and B must be equal in magnitude and opposite in direction.

$$F_{\mathrm{AB}} \cdot t_{\mathrm{AB}} = -F_{\mathrm{BA}} \cdot t_{\mathrm{BA}}$$

If we recall that impulse is equivalent to change in momentum, it follows that the change in momenta of the objects is equal but in the opposite directions. This can be equivalently expressed as the sum of the change in momenta being zero.

$$m_{\mathrm{A}} \cdot \Delta v_{\mathrm{A}} = -m_{\mathrm{B}} \cdot \Delta v_{\mathrm{B}}$$
$$m_{\mathrm{A}} \cdot \Delta v_{\mathrm{A}} + m_{\mathrm{B}} \cdot \Delta v_{\mathrm{B}} = 0$$

## WHAT IS INTERESTING ABOUT CONSERVATION OF MOMENTUM?

- Momentum is a vector quantity, and therefore we need to use vector addition when summing together the momenta of the multiple bodies which make up a system. Consider a system of two similar objects moving away from each other in opposite directions with equal speed. What is interesting is that the oppositely-directed vectors cancel out, so the momentum of the system as a whole is zero, even though both objects are moving.

- Collisions are particularly interesting to analyze using conservation of momentum. This is because collisions typically happen fast, so the time colliding objects spend interacting is short. A short interaction time means that the impulse, $F \cdot \Delta t$, dot, delta, t, due to external forces such as friction during the collision is very small.

- It is often easy to measure and keep track of momentum, even with complicated systems of many objects. Consider a collision between two ice hockey pucks. The collision is so forceful that it breaks one of the pucks into two pieces. Kinetic energy is likely not conserved in the collision, but momentum will be conserved.
  Provided we know the masses and velocities of all the pieces just after the collision, we can still use conservation of momentum to understand the situation. This is interesting because by contrast, it would be virtually impossible to use conservation of energy in this situation. It would be very difficult to work out exactly how much work was done in breaking the puck.

- Collisions with "immovable" objects are interesting. Of course, no object is really immovable, but some are so heavy that they appear to be. Consider the case of a bouncy ball of mass m*m*m traveling at velocity *v* towards a brick wall. It hits the wall and bounces back with velocity −*v*. The wall is

well attached to the earth and doesn't move, yet the momentum of the ball has changed by 2*mv* since velocity went from positive to negative.
If momentum is conserved, then the momentum of the earth and wall also must have changed by 2*mv*. We just don't notice this because the earth is so much heavier than the bouncy ball.