### 4.4 CONDITIONAL PROBABILITY

The word conditional is used to describe a probability that is dependent on some additional information given about an outcome or event.

Example 1: You randomly selects a letter from the word ACE. Find the probability that a randomly selected $\mathbf{E}$.

$$
P(\operatorname{select} E)=\frac{1}{3}
$$

Example 2: You randomly selects a letter from the word ACE. Find the probability that a randomly selected vowel.

$$
P(\text { select vowel })=\frac{2}{3}
$$

| The conditional $\quad$ probability |
| :--- | :--- |
| $P($ selects $E$, given that she selects a vowel $)=\frac{1}{2}$ |

Conditional probabilities are usually written using the symbol $\mid$ to mean given that.
We read $P(A \mid B)$ as 'the probability that $A$ occurs, given that $B$ occurs'.

Example 3: A child selected at random form a group of 11 boys and nine girls, and one of the girls is called Rose. Find the probability that Rose is selected given that a girls is selected.

$$
P(\text { Rose is selected } \mid \text { girl is selected })=\frac{1}{9}
$$

## Example 3:

The following table shows the numbers of students in a class who study Biology (B) and who study Chemistry (C).

|  | $\mathbf{B}$ | $\mathbf{B}^{\prime}$ | Totals |
| :---: | :---: | :---: | :---: |
| $\mathbf{C}$ | 9 | 8 | 17 |
| $\mathbf{C}^{\prime}$ | 7 | 1 | 8 |
| Totals | 16 | 9 | 25 |

Represent the data in a suitable Venn diagram, and find the probability that a randomly selected student:
a. Studies Chemistry, given that they study Biology
b. Does not study Biology, given that they do not study Chemistry.



Represent the data in a suitable Venn diagram, and find the probability that a randomly selected student:
a. Studies Chemistry, given that they study Biology
b. Does not study Biology, given that they do not study Chemistry.
a. $P(C \mid B)=\frac{9}{16}$
b. $P\left(B^{\prime} \mid C^{\prime}\right)=\frac{1}{8}$

Two children are selected at random from a group of five boys and seven girls. Find the probability that the second child is a boy, given that first child selected is:
a. A boy
b. A girl
a. $P($ second is a boy|first is a boy $)=\frac{4}{11}$
b. $P($ second is a boy|first is a girl $)=\frac{5}{11}$

1 One letter is randomly selected from the six letters in the word BANANA. Find the probability that:
a an N is selected, given that an A is not selected
b an A is selected, given that an N is not selected.

2 One hundred children were each asked whether they have brothers $(B)$ and whether they have sisters $(S)$. Their responses are given in the Venn diagram opposite.

Find the probability that a randomly selected child has:
a sisters, given that they have brothers
b brothers, given that they do not have sisters

c sisters or brothers, given that they do not have both.

3 Two photographs are randomly selected from a pack of 12 colour and eight black and white photographs.
Find the probability that the second photograph selected is colour, given that the first is:
a colour
b black and white.

4 The Venn diagram opposite shows the responses of 40 girls who were asked if they have an interest in a career in nursing $(N)$, dentistry $(D)$ or human rights $(H)$.
a Find the probability that a randomly selected girl has an interest in:
i human rights, given that she has an interest in nursing
ii nursing, given that she has no interest in dentistry.
b Describe any group of girls for whom dentistry is the least popular career of interest.


5 The quiz marks of 40 students are represented in the following bar chart.


Two students are selected at random from the group. Find the probability that the second student: a scored more than 5 , given that the first student did not score more than 5
b scored more than 7, given that the first student scored more than 7.

6 The histogram shown represents the times taken, in minutes, for 115 men to complete a task.

Two men are selected at random from the group. Find the probability that the:
a first man took less than 1minute, given that he took less than 3minutes
b second man took less than 6 minutes, given that the first man took less than 1 minute.


Time taken (min)

7 At an insurance company, $60 \%$ of the staff are male ( $M$ ) and $70 \%$ work full-time (FT). The following Venn diagram shows this and one other piece of information.
a What information is given by the value 0.10 in the Venn diagram?
b Find the value of $a$, of $b$ and of $c$.

c An employee is randomly selected. Find:
i $\mathrm{P}(M \mid F T)$
ii $\mathrm{P}\left(F T \mid M^{\prime}\right)$
iii $\mathrm{P}[(M \cap F T) \mid(M \cup F T)]$

8 Two fair triangular spinners, both with sides marked 1,2 and 3, are spun. Given that the sum of the two numbers spun is even, find the probability that the two numbers are the same.

9 Two ordinary fair dice are rolled and the two numbers rolled are added together to give the score. Given that a player's score is greater than 6 , find the probability that it is not greater than 8 .

9) $p(81 \mid 6 \uparrow)=\frac{11}{21}$

10 The circular archery target shown, on which $1,2,3$ or 5 points can be scored, is divided into four parts of unequal area by concentric circles. The radii of the circles are $3 \mathrm{~cm}, 9 \mathrm{~cm}, 15 \mathrm{~cm}$ and 30 cm .

You may assume that a randomly fired arrow pierces just one of the four areas and is equally likely to pierce any part of the target.
a Show that the probability of scoring 5 points is 0.01 .
b Find the probability of scoring 3 points, 2 points and 1 point with an arrow.
c Given that an arrow does not score 5 points, find the probability that it scores 1 point.
d Given that a total score of 6 points is obtained with two randomly fired arrows, find the probability that neither arrow scores 1 point.

c) $P\left(p p l(5 p)^{\circ}\right)=\frac{75}{99}=\frac{85}{33}$
d) $P\left(1^{\prime} \mid \sin 6\right)=\frac{1}{3}$


## 1. 2018 May/June p11

Vehicles approaching a certain road junction from town $A$ can either turn left, turn right or go straight on. Over time it has been noted that of the vehicles approaching this particular junction from town $A$, $55 \%$ turn left, $15 \%$ turn right and $30 \%$ go straight on. The direction a vehicle takes at the junction is independent of the direction any other vehicle takes at the junction.
(i) Find the probability that, of the next three vehicles approaching the junction from town $A$, one goes straight on and the other two either both turn left or both turn right.
[4]
(ii) Three vehicles approach the junction from town $A$. Given that all three drivers choose the same direction at the junction, find the probability that they all go straight on.


| i) | $P(\mathrm{SLL})=(0.3)(0.55)(0.55)=0.09075\left(\frac{363}{4000}\right)$ | MI | $\mathrm{P}(\mathrm{SLL}), \mathrm{P}(\mathrm{SKK}), \mathrm{P}(\mathrm{SSL})$ or $\mathrm{P}(\mathrm{SSK})$ seen |
| :---: | :---: | :---: | :---: |
|  | $P(\mathrm{SRR})=(0.3)(0.15)(0.15)=0.00675\left(\frac{27}{4000}\right)$ | Al | Two correct options 0.09075 or 0.00675 can be unsimplified |
|  | $\begin{aligned} \text { Total } & ={ }^{3} \mathrm{C}_{1} \times \mathrm{P}(\mathrm{SLL})+{ }^{3} \mathrm{C}_{1} \times \mathrm{P}(\mathrm{SRR}) \\ & =0.27225+0.02025 \end{aligned}$ | M1 | Summing 6 prob options not all identical |
|  | $\text { Prob }=0.293 \text { accept } 0.2925\left(\frac{117}{400}\right)$ | Al | Correct answer |
|  |  | 4 |  |
| ii) | $\mathrm{P}\left(\mathrm{SSS} \mid \text { all samedir }^{\mathrm{n}}\right)=\frac{P\left(\text { SSS and same dir }{ }^{n}\right)}{P(\text { same direction })}$ | B1 | $(0.3)^{3}$ oe seen on its own as num or denom of a fraction |
|  |  | M1 | Attempt at $\mathrm{P}(S S S+L L L+R R R)$ seen anywhere |
|  | $=\frac{0.3 \times 0.3 \times 0.3}{(0.15)^{3}+(0.55)^{3}+(0.3)^{3}}$ | Al | $(0.15)^{3}+(0.55)^{3}+(0.3)^{3}$ oe seen as denom of a fraction |
|  | $=0.137\left(\frac{108}{787}\right)$ | Al | Correct answer |
|  |  | 4 |  |

2. 2018 May/June p12

In a group of students, $\frac{3}{4}$ are male. The proportion of male students who like their curry hot is $\frac{3}{5}$ and the proportion of female students who like their curry hot is $\frac{4}{5}$. One student is chosen at random.
(i) Find the probability that the student chosen is either female, or likes their curry hot, or is both female and likes their curry hot.
(ii) Showing your working, determine whether the events 'the student chosen is male' and 'the student chosen likes their curry hot' are independent.


| Method 1 $\mathrm{P}(M \cap H)=\frac{3}{4} \times \frac{3}{5}=\frac{9}{20}(0.45)$ | B1 | Seen, accept unsimplified |
| :---: | :---: | :---: |
| $\mathrm{P}(F \text { or } M \cap H)=\frac{1}{4}+\frac{9}{20}=\frac{14}{20}$ | M1 | Numerical attempt at $\mathrm{P}(F)+\mathrm{P}(M \cap H)$ |
|  | Al | Correct unsimplified expression |
| $=\frac{7}{10}(0.7) \mathrm{OE}$ | Al | Correct final answer |
| Method 2 $\mathrm{P}\left(M \cap H^{\prime}\right)=\frac{3}{4} \times \frac{2}{5}=\frac{6}{20}(0.3)$ | B1 | Seen, accept unsimplified |
| $\mathrm{P}(F$ or $M \cap H)=1-\mathrm{P}\left(M \cap H^{\prime}\right)$ | M1 | Numerical attempt at $1-\mathrm{P}\left(M \cap H^{\prime}\right)$ |
| $=1-\frac{3}{4} \times \frac{2}{5}$ | Al | Correct unsimplified expression |
| $=\frac{7}{10}(0.7) \mathrm{OE}$ | Al | Correct final answer |


| 2(i) | Method 3 <br> $\mathrm{P}\left(F \cap H^{\prime}\right.$ or $\left.H\right)=\frac{1}{4} \times \frac{1}{5}+\frac{1}{4} \times \frac{4}{5}+\frac{3}{4} \times \frac{3}{5}$ | Bl | $\frac{3}{4} \times \frac{3}{5}\left(\frac{9}{20}\right)$ or $\frac{1}{4} \times \frac{4}{5}\left(\frac{4}{20}\right)$ or $\frac{3}{4} \times \frac{3}{5}+\frac{1}{4} \times \frac{4}{5}\left(\frac{13}{20}\right)$ seen |
| :--- | :--- | ---: | :--- |
|  | $=\frac{1}{20}+\frac{4}{20}+\frac{9}{20}$ | Ml | Numerical attempt at $\mathrm{P}\left(F \cap H^{\prime}\right)+\mathrm{P}(F \cap H)+\mathrm{P}(M \cap H)$ |
|  | $=\frac{7}{10}(0.7)$ oe | Al | Correct unsimplified expression |
| Method $4-$ Venn diagram style approach final answer <br> $\mathrm{P}(F \mathrm{U} H)=\mathrm{P}(F)+\mathrm{P}(H)-\mathrm{P}(F \cap H)$ |  |  |  |
| $=\frac{1}{4}+\frac{1}{4} \times \frac{4}{5}+\frac{3}{4} \times \frac{3}{5}-\frac{1}{4} \times \frac{4}{5}$ | Bl | $\frac{3}{4} \times \frac{3}{5}\left(\frac{9}{20}\right)$ or $\frac{1}{4} \times \frac{4}{5}\left(\frac{4}{20}\right)$ or $\frac{3}{4} \times \frac{3}{5}+\frac{1}{4} \times \frac{4}{5}\left(\frac{13}{20}\right)$ seen |  |
| $=\frac{1}{4}+\frac{4}{20}+\frac{9}{20}-\frac{4}{20}$ | Al | Numerical attempt at $\mathrm{P}(F)+\mathrm{P}(H)-\mathrm{P}(F \cap H)$ |  |
| $=\frac{7}{10}(0.7)$ oe | Al | Correct unsimplified expression final answer |  |


| 2(ii) | Method 1 $\begin{aligned} & (\mathrm{P}(M) \times \mathrm{P}(H)=) \frac{3}{4} \times \text { their } \frac{13}{20}=\frac{39}{80} \\ & (\mathrm{P}(M \cap H)=) \frac{3}{4} \times \frac{3}{5}=0.45 \end{aligned}$ | M1 | Unsimplified, or better, legitimate numerical attempt at $\mathrm{P}(M) \times \mathrm{P}(H)$ and $\mathrm{P}(M \cap H)$ <br> Descriptors $\mathrm{P}(M \cap H)$ and $\mathrm{P}(M) \times \mathrm{P}(H)$ seen, correct numerical evaluation and comparison, conclusion stated |
| :---: | :---: | :---: | :---: |
|  | $\frac{39}{80}(0.4875) \neq 0.45$, not independent | Al |  |
|  | Method 2 $\begin{aligned} \mathrm{P}(M \mid H) & =\frac{\mathrm{P}(M \cap H)}{\mathrm{P}(H)}=\frac{\frac{9}{20}}{\text { their } \frac{13}{20}}=\frac{9}{13} \\ \mathrm{P}(M) & =\frac{3}{4} \end{aligned}$ | M1 | Unsimplified, or better, numerical attempt at $\mathrm{P}(H)$ and $\mathrm{P}(M \cap H), \mathrm{P}(M)$ |
|  | $\frac{9}{13} \neq \frac{3}{4}$, not independent | Al | Descriptors $\mathrm{P}(M \cap H), \mathrm{P}(H)$ and $\mathrm{P}(M)$ OR $\mathrm{P}(M H)$ and $\mathrm{P}(M)$ seen, numerical evaluation and comparison, conclusion stated <br> Any appropriate relationship can be used, the M is awarded for an unsimplified, or better, numerical attempt at the terms required, the A mark requires the correct descriptors, numerical evaluation and comparison and the conclusion |
|  |  | 2 |  |

## 3. 2018 May/June p13

The members of a swimming club are classified either as 'Advanced swimmers' or 'Beginners'. The proportion of members who are male is $x$, and the proportion of males who are Beginners is 0.7 . The proportion of females who are Advanced swimmers is 0.55 . This information is shown in the tree diagram.


For a randomly chosen member, the probability of being an Advanced swimmer is the same as the probability of being a Beginner.
(i) Find $x$.
(ii) Given that a randomly chosen member is an Advanced swimmer, find the probability that the member is male.

| 3(i) | (1-x) and 0.45 (or 0.3 ) | B1 | Seen, either on tree diagram or elsewhere |
| :---: | :---: | :---: | :---: |
|  | Beginners: $0.7 \times x+{ }^{\prime} 0.45$ ' $\times{ }^{\prime}(1-x)^{\prime}=0.5$ <br> Or <br> Advanced: ' 0.3 ' $\times x+0.55 \times{ }^{\prime}(1-x)^{\prime}=0.5$ <br> Or $0.7 \times x+{ }^{\prime} 0.45{ }^{\prime} \times{ }^{\prime}(1-x)^{\prime}={ }^{\prime} 0.3^{\prime} \times x+0.55 \times{ }^{\prime}(1-x)$ ' | M1 | One of the three correct probability equations |
|  | $x=0.2$ oe | Al | Correct answer |
|  | Total: | 3 |  |
| 3(ii) | $\mathrm{P}(\mathrm{M} \mid \mathrm{A})=\frac{P(M \cap A)}{P(A)}=\frac{0.2 \times 0.3}{0.5}$ | M1 | 'i' $\times 0.3$ as num or denom of a fraction |
|  |  | M1 | $0.5\left(\right.$ or $\left(1-{ }^{\prime} \mathrm{i}^{\prime}\right) \times 0.55+\mathrm{i}$ ' $\times 0.3$ unsimplified) seen as denom of a fraction |
|  | $=0.12\left(\frac{3}{25}\right)$ | Al | Correct answer |
|  | Total: | 3 |  |

## 4. 2018 Oct/Nov p11

In a group of students, the numbers of boys and girls studying Art, Music and Drama are given in the following table. Each of these 160 students is studying exactly one of these subjects.

|  | Art | Music | Drama |
| :--- | :---: | :---: | :---: |
| Boys | 24 | 40 | 32 |
| Girls | 15 | 12 | 37 |

(i) Find the probability that a randomly chosen student is studying Music.
(ii) Determine whether the events 'a randomly chosen student is a boy' and 'a randomly chosen student is studying Music' are independent, justifying your answer.
(iii) Find the probability that a randomly chosen student is not studying Drama, given that the student is a girl.
(iv) Three students are chosen at random. Find the probability that exactly 1 is studying Music and exactly 2 are boys.

| 7(i) | $52 / 160=13 / 40,0.325$ | B1 | oe |
| :---: | :---: | :---: | :---: |
|  |  | 1 |  |
| 7(ii) | $\begin{aligned} & \mathrm{P}(\text { boy })=96 / 160: \mathrm{P}(\text { Music })=52 / 160 \\ & \mathrm{P}(\text { boy and } \text { Music })=40 / 160 \end{aligned}$ | M1 | Use of $\mathrm{P}(\mathrm{B}) \times \mathrm{P}(\mathrm{M})=\mathrm{P}(\mathrm{B} \cap \mathrm{M})$, appropriate probabilities used |
|  | $96 / 160 \times 52 / 160 \neq 40 / 160$ : Not independent | A1 | Numerical comparison and conclusion stated |
| 7(iii) | Method 1 |  |  |
|  | $\mathrm{P}($ not Music $/$ girl $)=\mathrm{P}($ not Music and girl) $/ \mathrm{P}($ girl $)$ $(27 / 160) /(64 / 160)$ | M1 | Appropriate probabilities in a fraction |
|  | $=\frac{27}{64}$ | Al | Correct answer www implies method |
|  | Method 2 |  |  |
|  | Direct from table | M1 | 27/a or $b / 64, a \neq 160$ |
|  | $\frac{27}{64}$ | Al | Correct answer www implies method |
|  |  | 2 |  |
| 7(iv) | $\mathrm{P}(\mathrm{B} \mathrm{M}) \times \mathrm{P}(\mathrm{B} \mathrm{NM}) \times \mathrm{P}(\mathrm{G} \mathrm{NM})$ or $\mathrm{P}(\mathrm{G} \mathrm{M}) \times \mathrm{P}(\mathrm{B} \mathrm{NM}) \times \mathrm{P}(\mathrm{B} \mathrm{NM})$ | M1 | One scenario identified with 3 probs multiplied |
|  | $40 / 160 \times 56 / 159 \times 52 / 158$ or $12 / 160 \times 56 / 159 \times 55 / 158$ | Al | One scenario correct (ignore multiplying factor) |
|  | $\times 3!$ | B1 | Both multiplying factors correct |
|  | $\begin{array}{ll} 0.17387 & 0.02759 \\ \mathrm{P}=0.17387+0.02759 & \end{array}$ | M1 | Both cases attempted and added (multiplying factor not required), accept unsimplified |
|  | $=0.201$ <br> Note: If score in this part is 0 , award SCB1 for $\frac{1}{160} \times \frac{1}{159} \times \frac{1}{158} \times k$, for positive integer $k$, seen | Al | Correct answer, oe |

## 5. 2018 Oct/Nov p12

Jake attempts the crossword puzzle in his daily newspaper every day. The probability that he will complete the puzzle on any given day is 0.75 , independently of all other days.
(i) Find the probability that he will complete the puzzle at least three times over a period of five days.
[3]

Kenny also attempts the puzzle every day. The probability that he will complete the puzzle on a Monday is 0.8 . The probability that he will complete it on a Tuesday is 0.9 if he completed it on the previous day and 0.6 if he did not complete it on the previous day.
(ii) Find the probability that Kenny will complete the puzzle on at least one of the two days Monday and Tuesday in a randomly chosen week.

| 3(i) | Method 1 |  |  |
| :---: | :---: | :---: | :---: |
|  | $\mathrm{P}(3)+\mathrm{P}(4)+\mathrm{P}(5)={ }^{5} \mathrm{C}_{3} 0.75^{3} \times 0.25^{2}+$ | M1 | One binomial term ${ }^{5} \mathrm{C}_{2} p^{x}(1-p)^{5-x}, x \neq 0$ or 5 , any $p$ |
|  | ${ }^{5} \mathrm{C}_{4} 0.75^{4} \times 0.25^{1}+{ }^{5} \mathrm{C}_{5} 0.75^{5} \times 0.25^{0}$ | M1 | Correct unsimplified expression |
|  | $\begin{aligned} =0.26367 & +0.39551+0.23730 \\ & =0.896(459 / 512) \end{aligned}$ | Al | Correct final answer, allow 0.8965 (isw) but not 0.897 alone |
|  | Method 2 |  |  |
|  | $1-\mathrm{P}(0)-\mathrm{P}(1)-\mathrm{P}(2)=1-{ }^{5} \mathrm{C}_{0} 0.75^{0} \times 0.25^{5}$ | M1 | One binomial term ${ }^{5} \mathrm{C}_{4} p^{x}(1-p)^{5-x}, x \neq 0$ or 5 , any $p$ |
|  | $-{ }^{5} \mathrm{C}_{1} 0.755^{1} \times 0.25^{4}-{ }^{5} \mathrm{C}_{2} 0.75^{2} \times 0.25^{3}$ | M1 | Correct simplified expression |
|  | $\begin{aligned} & =1-0.00097656-0.014648-0.087891 \\ & =0.896(459 / 512) \end{aligned}$ | Al | Correct final answer, allow 0.8965 (isw) but not 0.897 alone |
|  |  | 3 |  |
| $\cdots$ | ........ |  | - $\ldots \ldots \ldots$ |
| 3(ii) | Method 1 |  |  |
|  | $\begin{aligned} & \mathrm{P}(\mathrm{C}, \mathrm{C})+\mathrm{P}\left(\mathrm{C}, \mathrm{C}^{\prime}\right)+\mathrm{P}\left(\mathrm{C}^{\prime}, \mathrm{C}\right) \\ & 0.8 \times 0.9 \end{aligned}$ | B1 | Unsimplified prob completed on both days |
|  | $0.8 \times 0.1+0.2 \times 0.6$ | M1 | Unsimplified prob $0.8 \times a+0.2 \times b, a=0.1$ or $0.4, b=0.6$ or 0.9 |
|  | $=0.92$ oe | Al | Correct final answer |
|  | Method 2 |  |  |
|  | $1-\mathrm{P}\left(\mathrm{C}^{\prime}, \mathrm{C}^{\prime}\right)=1-0.2 \times 0.4$ | B1 | Unsimplified prob completed on no days |
|  |  | M1 | $1-0.2 \times a, a=0.1$ or 0.4 allow unsimplified |
|  | $=0.92$ | Al | Correct final answer |
|  |  | 3 |  |

## 6. 2018 Oct/Nov p13

A box contains 3 red balls and 5 blue balls. One ball is taken at random from the box and not replaced. A yellow ball is then put into the box. A second ball is now taken at random from the box.
(i) Complete the tree diagram to show all the outcomes and the probability for each branch.
(ii) Find the probability that the two balls taken are the same colour.
(iii) Find the probability that the first ball taken is red, given that the second ball taken is blue.

| 3(i) |
| :--- |

## 7. 2019 May/June p11

Jameel has 5 plums and 3 apricots in a box. Rosa has $x$ plums and 6 apricots in a box. One fruit is chosen at random from Jameel's box and one fruit is chosen at random from Rosa's box. The probability that both fruits chosen are plums is $\frac{1}{4}$. Write down an equation in $x$ and hence find $x$. [3]
2.

A fair six-sided die is thrown twice and the scores are noted. Event $X$ is defined as "The total of the two scores is 4'. Event $Y$ is defined as "The first score is 2 or 5'. Are events $X$ and $Y$ independent? Justify your answer.

| 2 | 5 Resa P (plum) $=\frac{x}{x+6}$ | M1 | Their 2 probabilities for P (plum) multiplied and equated to $1 / 4$ |
| :---: | :---: | :---: | :---: |
|  | $\frac{5}{8} \times \frac{x}{x+6}=\frac{1}{4}$ | Al | Correct equation oe |
|  | $(x=4$ | Al | SC correct answer with no appropriate equations i.e. common sense B1 |
|  |  | 3 |  |


| 3 | $\mathrm{P}(\mathrm{X})=\frac{3}{36}\left(\frac{1}{12} o e\right)$ | Bl |  |
| :--- | :--- | ---: | :--- |
|  | $\mathrm{P}(\mathrm{Y})=\frac{12}{36}\left(\frac{1}{3} o e\right)$ | Bl |  |
|  | $\mathrm{P}(\mathrm{X} \cap \mathrm{Y})=\frac{1}{36}$ | Ml | Independent method to find $\mathrm{P}(X \cap Y)$ without multiplication, either <br> stated or by listing or circling numbers on a probability space <br> diagram. OR condititional prob with a single fraction numerator |
|  | $\mathrm{P}(\mathrm{X}) \times \mathrm{P}(\mathrm{Y})=\mathrm{P}(\mathrm{X} \cap \mathrm{Y})$, independent | Al | Numerical comparison and conclusion, www |

## 8. 2019 May/June p12

Two ordinary fair dice are thrown and the numbers obtained are noted. Event $S$ is The sum of the numbers is even'. Event $T$ is 'The sum of the numbers is either less than 6 or a multiple of 4 or both'. Showing your working, determine whether the events $S$ and $T$ are independent.

| $\mathrm{P}(\mathrm{S})=\frac{1}{2}$ | Bl |  |
| :--- | ---: | :--- |
| $\mathrm{P}(T)=\frac{16}{36}\left(\frac{4}{9}\right)$ | Bl |  |
| $\mathrm{P}(\mathrm{S} \cap T)=\frac{10}{36}\left(\frac{5}{18}\right)$ | Ml | $\mathrm{P}(\mathrm{S} \cap T)$ found by multiplication scores M0 <br> M1 awarded if their value is identifiable in their sample space diagram or <br> Venn diagram or list of terms or probability distribution table (oe) |
| $\mathrm{P}(S) \mathrm{P}(T) \neq \mathrm{P}(S \cap T)$ so not independent | Al | $8 / 36,10 / 36 \mathrm{P}(\mathrm{S}) \times \mathrm{P}(\mathrm{T})$ and $\mathrm{P}(\mathrm{S} \cap T)$ seen in workings and correct <br> conclusion stated, www |

## 9. 2019 May/June p13

Megan sends messages to her friends in one of 3 different ways: text, email or social media. For each message, the probability that she uses text is 0.3 and the probability that she uses email is 0.2 . She receives an immediate reply from a text message with probability 0.4 , from an email with probability 0.15 and from social media with probability 0.6 .
(i) Draw a fully labelled tree diagram to represent this information.
[2]
(ii) Given that Megan does not receive an immediate reply to a message, find the probability that the message was an email.


| 2(ii) | $\mathrm{P}($ email $\mid N R)=\frac{\mathrm{P}(\mathrm{email} \cap \mathrm{NR})}{\mathrm{P}(\mathrm{NR})}=\frac{0.2 \times 0.85}{0.3 \times 0.6+0.2 \times 0.85+0.5 \times 0.4}$ | Ml | $\mathrm{P}(\mathrm{email}) \times \mathrm{P}(\mathrm{NR})$ seen as numerator of a fraction, consistent with <br> their tree diagram |
| :--- | ---: | ---: | :--- |
|  | $=\frac{0.17}{0.18+0.17+0.2}=\frac{0.17}{0.55}$ | $\mathbf{M l}$ | Summing three appropriate 2-factor probabilities, consistent with <br> their tree diagram, seen anywhere <br> 0.55 oe (can be unsimplified) seen as denom of a fraction |
|  | $=0.309, \frac{17}{55}$ | Al |  |
|  | $\mathbf{A l}$ | Correct answer |  |

