Probability & Statistics

Chapter 4
Probability

Cambridge AS level

4.1 EXPERIMENTS, EVENTS AND OUTCOMES

The result of an experiment is called an **outcome** or **elementary event**, and a combination of these is known simply as an event.

Rolling an ordinary fair die is an experiment that has six possible outcomes: 1, 2, 3, 4, 5 or 6.

Obtaining an odd number with the die is an event that has three **favourable** outcomes: 1, 3 or 5.

Random selection and equiprobable events

The purpose of selecting object at **random** is to ensure that each has the same chance of being selected. This method of selection is called **fair** or **unbiased**, and the selection of any particular object is said to be **equally likely** or **equiprobable**.

Bais/Unfair

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KEY POINT:
When one object is randomly selected from n objects,
$$P(\textit{selecting any particular object}) = \frac{1}{n}$$

Random selection and equiprobable events

The probability that an event occurs is equal to the proportion of equally likely outcomes that are favourable to the event.

KEY POINT:

$$P(event) = \frac{Number\ of\ favourable\ equally\ likely\ outcomes}{Total\ number\ of\ equally\ likely\ outcomes}$$

Consider randomly selecting 1 student from a group of 19, where 11 are boys and 8 are girls.

Event/Outcome	Probability	Description
Selecting any particular boy	$\dfrac{1}{19}$	These three outcome are equally likely.
Selecting any particular girl	$\dfrac{1}{19}$	
Selecting any particular student	$\dfrac{1}{19}$	
Selecting a boy	$\dfrac{11}{19}$	11 of the 19 equally likely outcomes are favorable to this event.
Selecting a girl	$\dfrac{8}{19}$	8 of the 19 equally likely outcomes are favourable to this event.

The word **particular** species one object. It does not matter whether that object is a boy, a girl or a student.

Exhaustive events

A set of events that contains all the possible outcomes of an experiment is said to be exhaustive. In the special case of event A and its **complement**, not A, the sum of their probabilities is 1 because one them is certain to occur. Recall that the notation used for complement of set A is A'.

KEY POINT:

$$P(A) + P(not\ A) = 1\ or\ P(A) + P(A') = 1$$

Example 2: Copy and the complete.

Experiment	Exhaustive events		Probabilities
	A	A$'$	
Toss a fair coin	Heads	Tails	$\dfrac{1}{2} + \dfrac{1}{2} = 1$
Roll a fair die	Less than 2	2 or more	$\dfrac{1}{6} + \dfrac{5}{6} = 1$
Play a game of chess	Win	Not win	$\dfrac{1}{2} + \dfrac{1}{2} = 1$

Trails and expectation

Each repeat of an experiment is called **a trial**. The proportion of trails in which an event occurs is its **relative frequency**, and we can use this an estimate of the probability that the event occurs.

KEY POINT:

In n trials, event A is expected to occur $n \times P(A)$ times.

If we know the probability of an event occurring, we can estimate the number of times it is likely to occur in a series of trials. This is a statement of our **expectation**.

The probability of rain on any particular day in mountain village is 0.2. On how many days is rain not excepted in a year of 365 days?

$$P(does\ not\ rain) = 1 - 0.2 = 0.8$$

$$365 \times 0.8 = 292$$

EXERCISE 4A

1   A teacher randomly selects one student from a group of 12 boys and 24 girls. Find the probability that the teacher selects:

   a   a particular boy   **b**   a girl.

2   United's manager estimates that the team has a 65% chance of winning any particular game and an 85% chance of not drawing any particular game.

   a   What are the manager's estimates most likely to be based on?

   b   If the team plays 40 games this season, find the manager's expectation of the number of games the team will lose.

   c   If the team loses one game more than the manager expects this season, explain why this does not necessarily mean that they performed below expectation.

3   Katya randomly picks one of the 10 cards shown.

A	C	3	C	B	A	C	C	B	C

   If she repeats this 40 times, how many times is Katya expected to pick a card that is not blue and does not have a letter B on it?

4   A numbered wheel is divided into eight sectors of equal size, as shown. The wheel is spun until it stops with the arrow pointing at one of the numbers.

   Axel decides to spin the wheel 400 times.
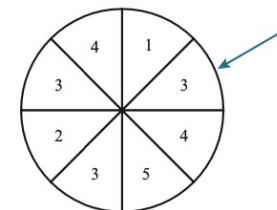
   a   Find the number of times the arrow is not expected to point at a 4.

   b   How many more times must Axel spin the wheel so that the expected number of times that the arrow points at a 4 is at least 160?

5   A bag contains black and white counters, and the probability of selecting a black counter is $\frac{1}{6}$.

   a   What is the smallest possible number of white counters in the bag?

   b   Without replacement, three counters are taken from the bag and they are all black. What is the smallest possible number of white counters in the bag?

6   When a coin is randomly selected from a savings box, each coin has a 98% chance of not being selected. How many coins are in the savings box?

7   A set of data values is 8, 13, 17, 18, 24, 32, 34 and 38. Find the probability that a randomly selected value is more than one standard deviation from the mean.

8   One student is randomly selected from a school that has 837 boys. The probability that a girl is selected is $\frac{4}{7}$. Find the probability that a particular boy is selected.

⏪ REWIND

We studied the mean in Chapter 2, Section 2.2 and standard deviation in Chapter 3, Section 3.3.