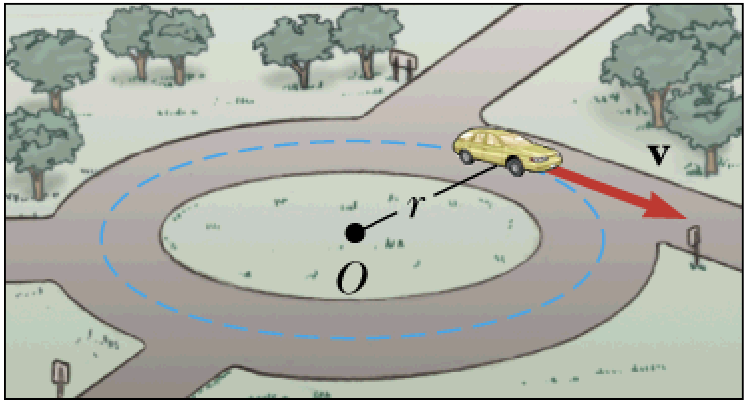
Circular motion

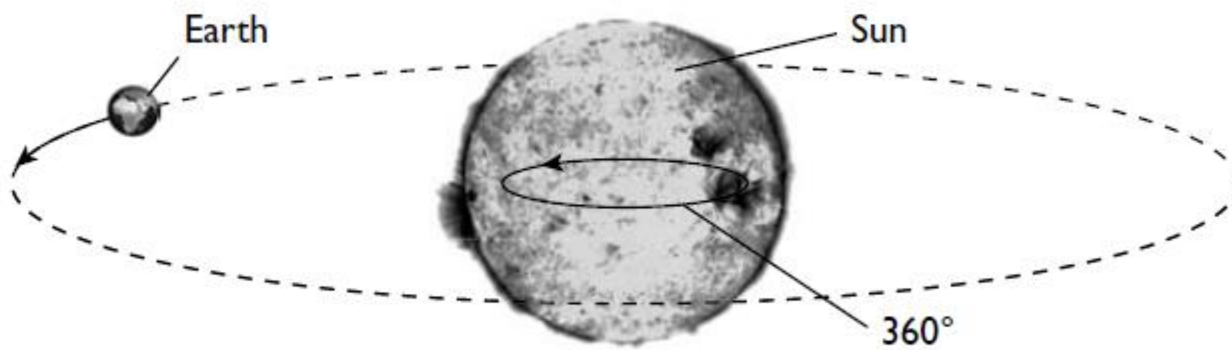
Circular Motion … Examples

- Earth moving around the Sun
- Electron moving around a proton in a hydrogen atom.
- Moving car in a roundabout.
- Simple pendulum.



Circular motion

You are familiar with the use of degrees to measure angles, with a complete circle equal to 360°. There is no real reason why a circle is split into 360° — it probably arises from the approximate number of days it takes for the Earth to orbit the Sun.

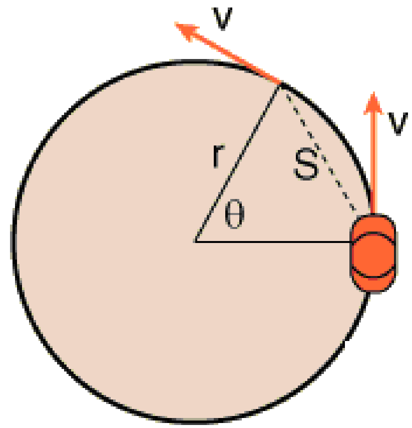


It is much more convenient to use radians, where the angle in radians is the ratio of the arc length to the radius:

angle (in radians) = arc length/radius

Circular motion

The angle measured, θ, measured in radians, is define by the following equation :



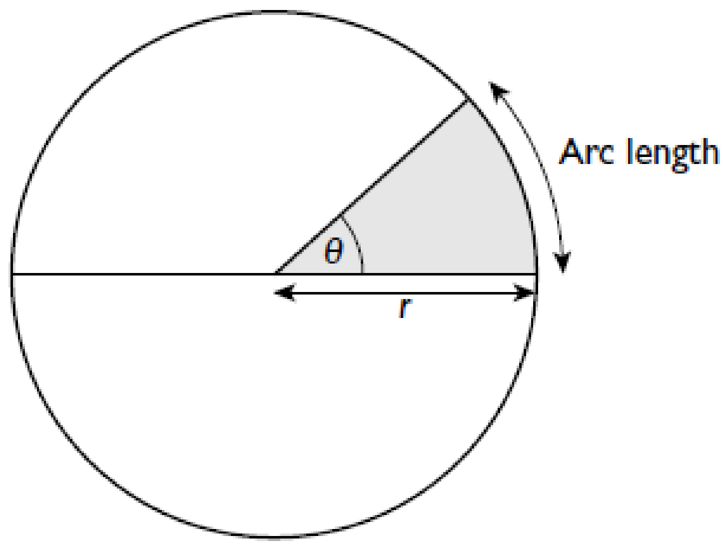
$$\theta = \frac{s}{r}$$

Definitions

One radian is that angle supported by an arc length in a circle equal to the radius of the circle.

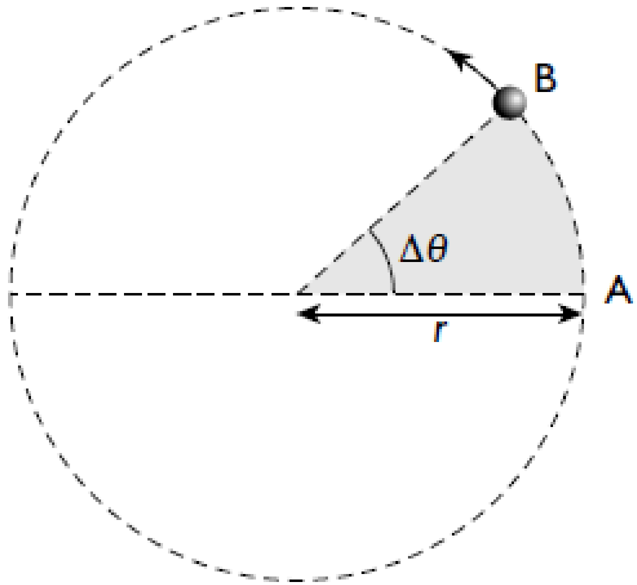
Circular motion

The circumference of a circle = $2\pi r$, where r is the radius. Hence, the angle subtended by a complete circle (360°) = $2\pi r/r = 2\pi$. 360° = 2π radians. This can be expressed as 1° = $2\pi/360$ radians or 1 radian = $(360/2\pi)°$.



Angular displacement

Consider a particle moving at constant speed (*v*) round a circle.



<u>Angular displacement is defined as the change in angle</u> (measured in radians).

Angular velocity

As the particle moves round the circle, the angular displacement increases at a steady rate. The rate of change in angular displacement is called the angular velocity (ω). Angular velocity is therefore defined as the change in angular displacement per unit time:

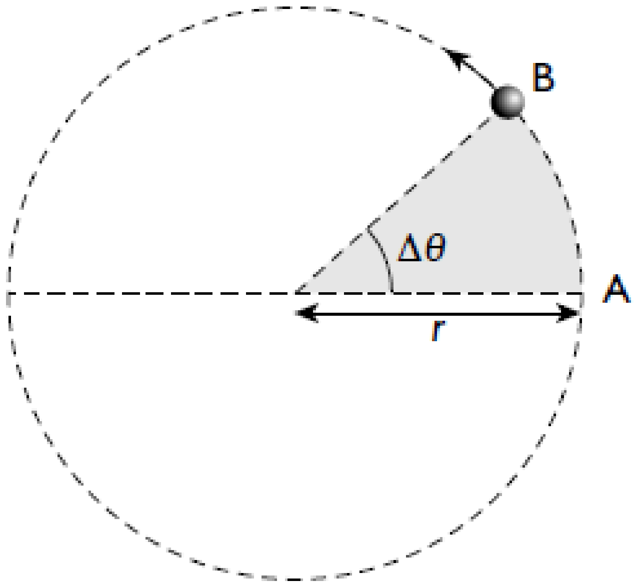
$$\omega = \Delta\theta/\Delta t$$

Comparison with translational motion

Many of the concepts we met in kinematics at AS have their equivalent in circular motion. This is shown in Table.

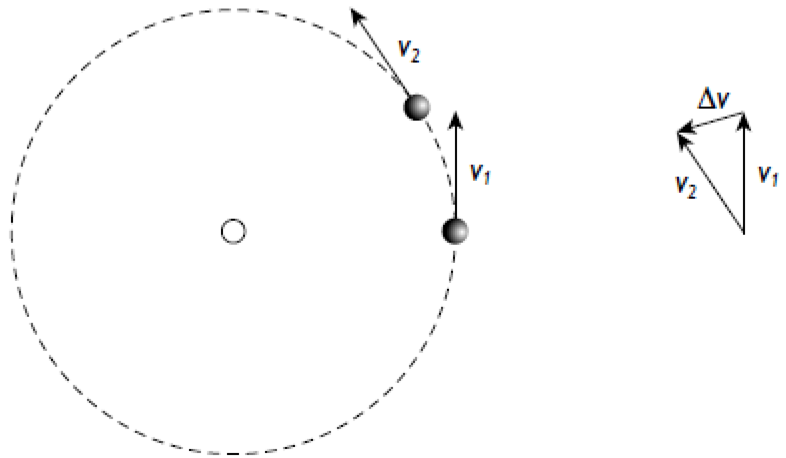
Translational motion			**Circular motion**		
Quantity	Unit	Relationships	Quantity	Unit	Relationships
Displacement (s)	m		Angular displacement (θ)	rad	
Velocity (v)	$m\,s^{-1}$	$v = \Delta s/\Delta t$	Angular velocity (ω)	$rad\,s^{-1}$	$\omega = \Delta\theta/\Delta t$

Relationship between angular velocity and speed



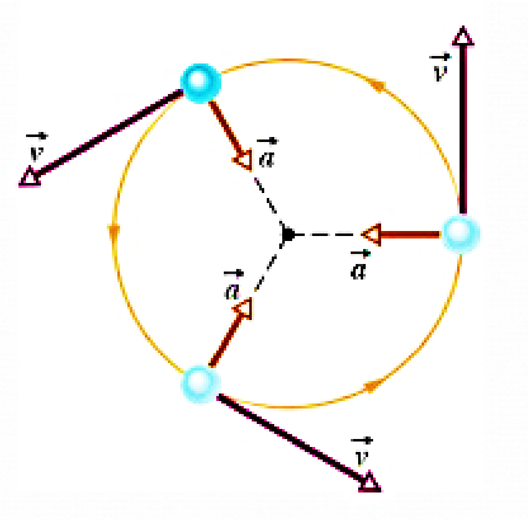
$\omega = \Delta\theta/\Delta t$, but $\Delta\theta = AB/r$. Therefore $\omega = AB/r\Delta t$, $AB/\Delta t$ = distance travelled/time = v thus $\omega = v/r$ or, rearranging the formula, $\boldsymbol{v} = \omega \boldsymbol{r}$

Acceleration in circular motion at constant speed



Consider a particle moving round a circle. At time t it has a velocity of v_1. After a short interval of time, Δt, it has the velocity v_2 — the same magnitude, but the direction has changed. The vector diagram shows the change of velocity Δv. You can see that this is towards the centre of the circle, the acceleration being $\Delta v/\Delta t$. As the body moves round the circle, the direction of its velocity is continuously changing, the change always being towards the centre of the circle.

Acceleration in circular motion at constant speed



Thus the particle has an acceleration of constant magnitude but whose direction is always towards the centre of the circle.

Such an acceleration is called a **centripetal acceleration**. The magnitude of the acceleration is given by the formulae:

$$a = \frac{\upsilon^2}{r} \text{ and } a = \omega^2 \boldsymbol{r}$$

Centripetal force and acceleration

- When an object moves in a circle, it must **<u>accelerates</u>**.
- The acceleration directs toward the centre of the circle.
- According to Newton's 2nd law, there has to be a **<u>force</u>** to produce such acceleration
- This force must point toward the centre of the circle (**<u>Centripetal Force</u>**)

$$F = ma = m\frac{\upsilon^2}{r} \text{ and } \boldsymbol{F} = \boldsymbol{m}\mathrm{a} = m\omega^2\boldsymbol{r}$$

Origin of Centripetal Force

Circular Motion	Centripetal Force
Satellite in orbit around Earth	Gravitational force of the Earth
Car moving around a flat-curve	Static frictional force
Car moving around a banked-exit	Static frictional force and normal force
Toy-plane tied to a rope and moving in a circle	Tension in the rope
Astronaut in a rotating space station	Normal force by the surface/floor
Rider at a roller coaster	weight and/or normal force