Probability & Statistics

Chapter 4

Probability

Cambridge AS level

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4.2 INDEPENDENT EVENTS AND THE MULTIPLICATION LAW

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Two events are said to be **independent** if either can occur without being affected by the occurrence of the other.

Examples of this are making selections with replacement and performing separate actions, such as rolling two dice.

KEY POINT:

The multiplication law for independent events is
$$P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B).$$

This can be extended for any number of independent events:
$$P(A \text{ and } B \text{ and } C \text{ and } \dots) = P(A \cap B \cap C \cap \cdots)$$
$$= P(A) \times P(B) \times P(C) \dots$$

Example 1: Consider the following bag, which contains two blue balls (B) and five white balls (W). We will select one ball at random, replace it and then select another ball.

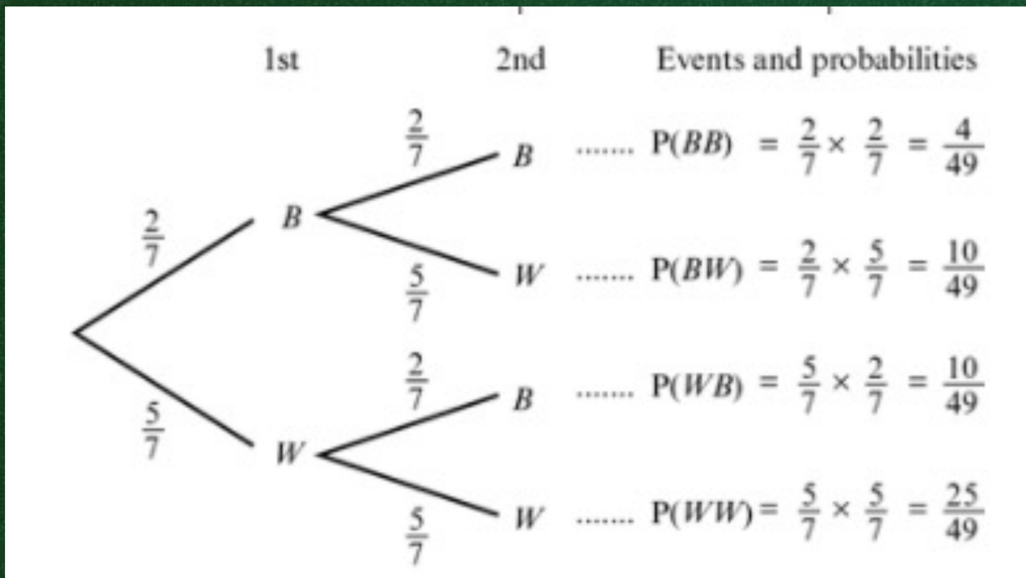
For the **first selection:**

$$P(B) = \frac{2}{7} \ and \ P(W) = \frac{5}{7}$$

For the **second selection:**

$$P(B) = \frac{2}{7} \ and \ P(W) = \frac{5}{7}$$

Example 1: Consider the following bag, which contains two blue balls (B) and five white balls (W). We will select one ball at random, replace it and then select another ball.



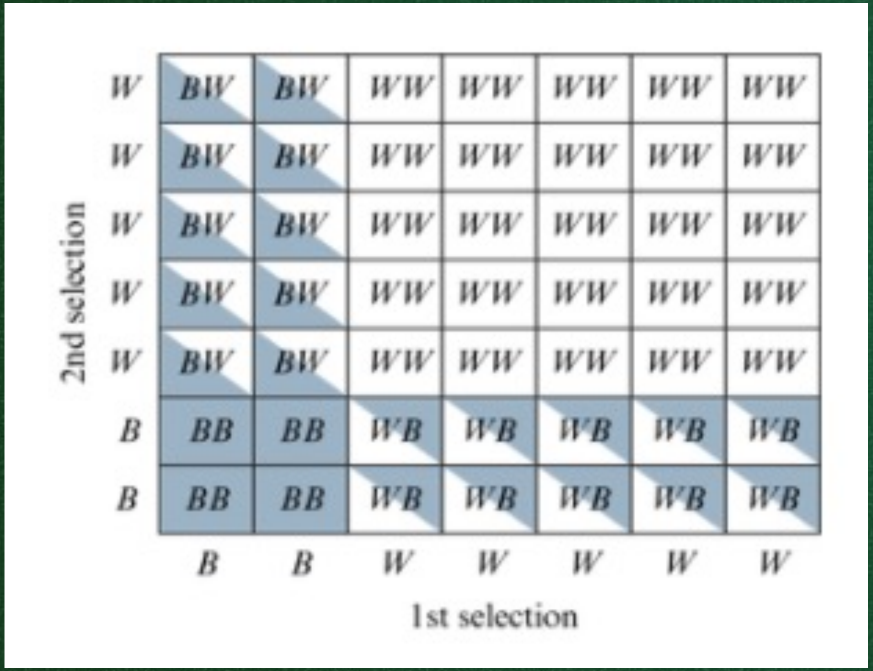
TIP: We can denote the event '2 blue balls are selected' by
BB;
B and B;
B&B or
B, B.

TIP: Events BB, BW, WB, and WW are **exhaustive**, so their probabilities sum to 1.

TIP: The first and second selections are made from the same seven balls, so probabilities are identical and independent.

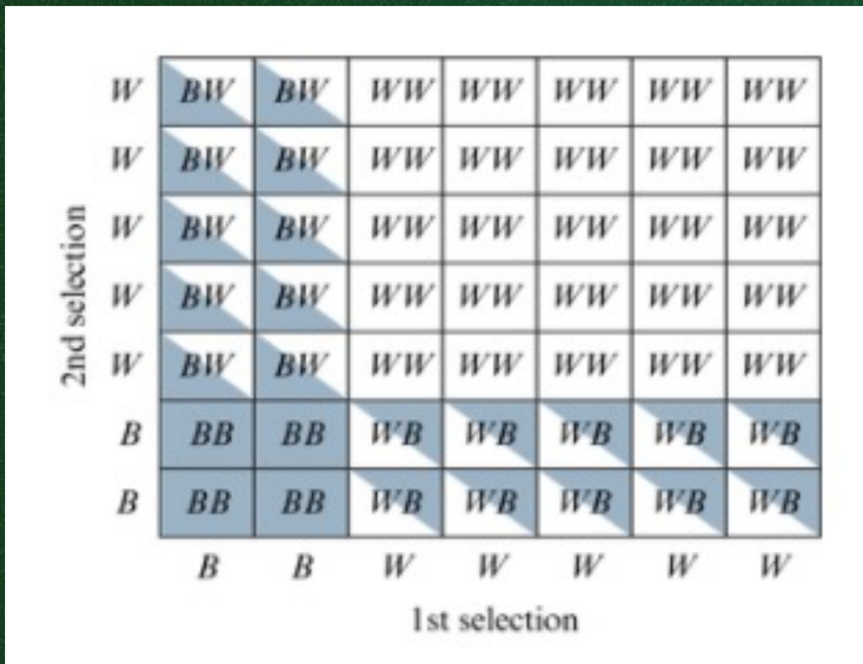
Example 1: Consider the following bag, which contains two blue balls (B) and five white balls (W). We will select one ball at random, replace it and then select another ball.

The diagram shows the $7 \times 7 = 49$ equally likely outcomes and the four mutually exclusive combined events BB, BW, WB *and* WW.



TIP: If we just used a 2 by 2 diagram, with B and W as the outcomes of each selection, we could not just count cells to find probabilities, because the events in those four cells would not be equally likely.

Example 1: Consider the following bag, which contains two blue balls (B) and five white balls (W). We will select one ball at random, replace it and then select another ball.



$$P(at\ least\ 1\ blue)$$
$$= P(BB) + P(BW) + P(WB)$$
$$= \frac{4}{49} + \frac{10}{49} + \frac{10}{49} = \frac{24}{49}$$

Alternatively,

$$P(at\ least\ 1\ blue) = 1 - P(WW)$$
$$= 1 - \frac{25}{49} = \frac{24}{49}$$

TIP: Probabilities are equal to the relative frequencies of the favourable outcomes.

Example 2: Find the probability that the sum of the scores on three rolls of an ordinary fair die is less than 5.

P(sum<5)=P(4)+P(3)

(1; 1; 1) (1; 1; 2) (2; 1; 1) (1; 2; 1)

$$P(3)=\frac{1}{216} \qquad P(4)=\frac{3}{216}$$

$$P(sum<5)=P(4)+P(3)$$

$$=\frac{3}{216}+\frac{1}{216}=\frac{4}{216}=\frac{1}{54}$$

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

	1	2	3	4	5	6
2	3	4	5	6	7	8
3	4	5	6	7	8	9
3	4	5	6	7	8	9
4	5	6	7	8	9	10
4	5	6	7	8	9	10
4	5	6	7	8	9	10

Example 3: Abha passes through three independent sets of traffic lights when she drives to work. The probability that she has to stop at any particular set of lights is 0.2. Find the probability that Abha:

a. First has to stop at the second set of lights.

b. Has to stop at exactly one set of lights.

c. Has to stop at any set of lights.

a. Stop => S $P(S) = 0.2$

Not Stop =>G $P(G) = 0.8$

$$P(GSS) + P(GSG) = 0.8\times0.2\times0.2 + 0.8\times0.2\times0.8$$

$$= 0.8\times0.2\times(0.2 + 0.8)$$

$$= 0.16$$

Example 3:

Abha passes through three independent sets of traffic lights when she drives to work. The probability that she has to stop at any particular set of lights is 0.2. Find the probability that Abha:

a. First has to stop at the second set of lights.

b. Has to stop at exactly one set of lights.

c. Has to stop at any set of lights.

b. Stop => S $P(S) = 0.2$

Not Stop =>G $P(G) = 0.8$

$P(has\ to\ stop\ at\ exactly\ 1\ set\ of\ lights) =$

$P(SGG) + P(GSG) + P(GGS) =$

$0.2\times0.8\times0.8 + 0.8\times0.2\times0.8 + 0.8\times0.8\times0.2 = 0.384$

Example 3: Abha passes through three independent sets of traffic lights when she drives to work. The probability that she has to stop at any particular set of lights is 0.2. Find the probability that Abha:

a. First has to stop at the second set of lights.

b. Has to stop at exactly one set of lights.

c. Has to stop at any set of lights.

c. Stop => S $P(S) = 0.2$

Not Stop =>G $P(G) = 0.8$

$$P(has\ to\ stop\ ) = 1 - P(does\ not\ have\ to\ stop)$$
$$= 1 - P(GGG)$$
$$= 1 - 0.8^3$$
$$= 0.488$$

Homework

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EXERCISE 4C

1 Using a tree diagram, find the probability that exactly one head is obtained when two fair coins are tossed.

2 Two ordinary fair dice are rolled. Using a possibility diagram, find the probability of obtaining:

a two 6s **b** two even numbers **c** two numbers whose product is 6.

3 It is known that 8% of all new FunX cars develop a mechanical fault within a year and that 15% independently develop an electrical fault within a year. Find the probability that within a year a new FunX car develops:

a both types of fault **b** neither type of fault.

4 A certain horse has a 70% chance of winning any particular race. Find the probability that it wins exactly one of its next two races.

5 The probabilities that a team wins, draws or loses any particular game are 0.6, 0.1 and 0.3, respectively.

a Find the probability that the team wins at least one of its next two games.

b If 2 points are awarded for a win, 1 point for a draw and 0 points for a loss, find the probability that the team scores a total of more than 1 point in its next two games.

6 On any particular day, there is a 30% chance of snow in Slushly. Find the probability that it snows there on:

a none of the next 3 days **b** exactly one of the next 3 days.

7 Fatima will enter three sporting events at the weekend.
Her chances of winning each of them are shown in the following table.

Event	Shot put	Javelin	Discus
Chance of winning	85%	40%	64%

a Assuming that the three events are independent, find the probability that Fatima wins:

i the shot put and discus **ii** the shot put and discus only **iii** exactly two of these events.

b What does 'the three events are independent' mean here? Give a reason why this may not be true in real life.

8 A fair six-sided spinner, P, has edges marked 0, 1, 2, 2, 3 and 4.

A fair four-sided spinner, Q, has edges marked 0, –1, –1 and –2.

Each spinner is spun once and the numbers on which they come to rest are added together to give the score, S. Find:

a $P(S = 2)$ **b** $P(S^2 = 1)$

9 Letters and packages can take up to 2 days to be delivered by Speedipost couriers. The following table shows the percentage of items delivered at certain times after sending.

	Same day	After 1 day	After 2 days
Letter	40%	50%	10%
Package	15%	55%	30%

a Is there any truth in the statement 'If you post 10 letters on Monday then only nine of them will be delivered before Wednesday'? Give a reason for your answer.

b Find the probability that when three letters are posted on Monday, none of them are delivered on Tuesday.

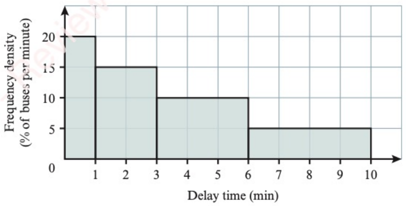
c Find the probability that when a letter and a package are posted together, the letter arrives at least 1 day before the package.

10 The following histogram represents the results of a national survey on bus departure delay times.

Two buses are selected at random. Calculate an estimate of the probability that:

a both departures were delayed by less than 4 minutes

b at least one of the buses departed more than 7 minutes late.



11 Praveen wants to speak on the telephone to his friend. When his friend's phone rings, he answers it with constant probability 0.6. If Praveen's friend doesn't answer his phone, Praveen will call later, but he will only try four times altogether. Find the probability that Praveen speaks with his friend:

a after making fewer than three calls **b** on the telephone on this occasion.

Homework

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12 Each morning, Ruma randomly selects and buys one of the four newspapers available at her local shop. Find the probability that she buys:

a the same newspaper on two consecutive mornings

b three different newspapers on three consecutive mornings.

13 A coin is biased such that the probability that three tosses all result in heads is $\frac{125}{512}$. Find the probability of obtaining no heads with three tosses of the coin.

14 In a group of five men and four women, there are three pairs of male and female business partners and three teachers, where no teacher is in a business partnership. One man and one woman are selected at random. Find the probability that they are:

a both teachers

b in a business partnership with each other

c each in a business partnership but not with each other.

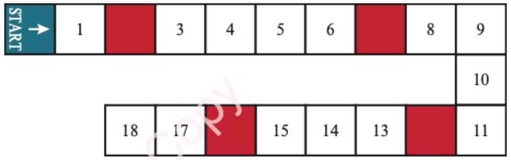
15 A biased die in the shape of a pyramid has five faces marked 1, 2, 3, 4 and 5. The possible scores are 1, 2, 3, 4 and 5 and $P(x) = \frac{k - x}{25}$, where k is a constant.

a Find, in terms of k, the probability of scoring:

i 5 **ii** less than 3.

b The die is rolled three times and the scores are added together. Evaluate k and find the probability that the sum of the three scores is less than 5.

16 A game board is shown in the diagram.



Players take turns to roll an ordinary fair die, then move their counters forward from 'start' a number of squares equal to the number rolled with the die. If a player's counter ends its move on a coloured square, then it is moved back to the start.

a Find the probability that a player's counter is on 'start' after rolling the die:

i once **ii** twice.

b Find the probability that after rolling the die three times, a player's counter is on:

i 18 **ii** 17