4.2 MUTUALLY EXCLUSIVE EVENTS AND THE ADDITION LAW

To find the probability that event A or event B occurs, we can simply add the probabilities of the two events together, but only if $A\ and\ B$ are **mutually exclusive.**

Mutually exclusive events have **no common favourable outcomes**, which means that it is not possible for both events to occur, so $\mathbf{P(A\ and\ B) = 0}$

Example 1:

When we roll an ordinary die, the events $even\ number = \{2; 4; 6\}$ and $factor\ of\ 5 = \{1; 5\}$ are mutually exclusive because they have no common outcomes.

We can that the intersection of these two sets is empty, Therefore

$$P(even\ or\ factor\ of\ 5) = P(even) + P(factor\ of\ 5)$$

Events are **not mutually exclusive** if they have **at least one common favourable outcome**, which means that it is possible for both events to occur, so $\boldsymbol{P(A\ and\ B) \neq 0}$

Example 2:

When we roll an ordinary die, the events $odd\ number = \{1;3;5\}$ and $factor\ of\ 5 = \{1;5\}$ are not mutually exclusive because they have common favourable outcomes.

We can that the intersection of these two sets is not empty, Therefore

$$P(odd\ or\ factor\ of\ 5) \neq P(odd) + P(factor\ of\ 5)$$

The addition law for mutually exclusive events is

$$\boldsymbol{P(A\ or\ B) = P(A) + P(B).}$$

This can be extended for any number of mutually exclusive events:

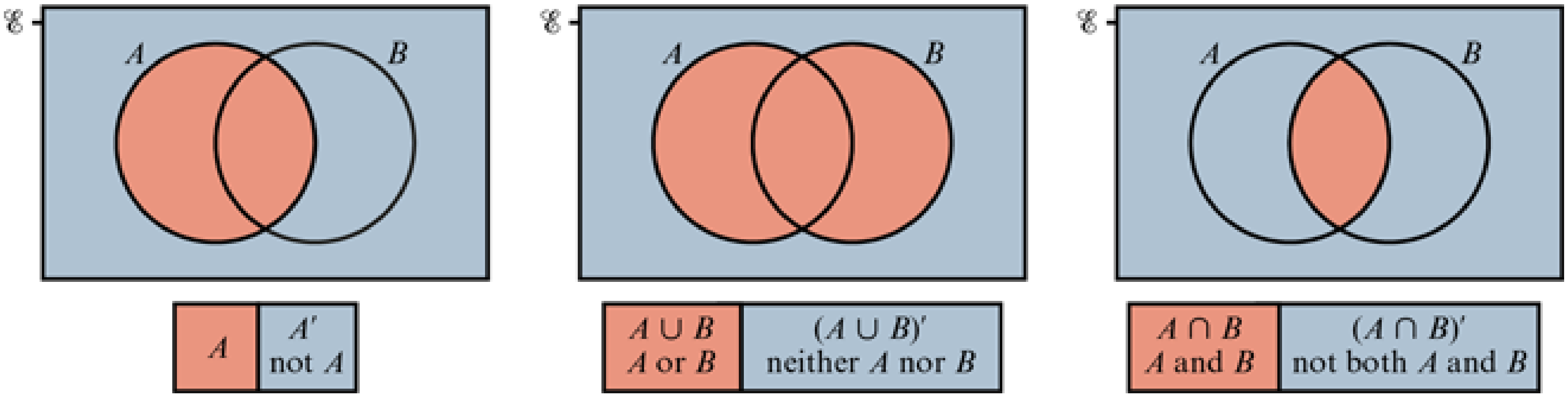
$$\boldsymbol{P(A\ or\ B\ or\ C\ or\ \ldots) = P(A) + P(B) + P(C) + \cdots}$$

Venn diagrams

Venn diagrams are useful tools for solving problems in probability. We can use them to show favourable outcomes or the number of favourable outcomes or the probabilities of particular events.

The number of outcomes favourable to event A is denoted by $n(A)$.

The set of outcomes that are not favourable to event A is the complement of A, denoted by A'.



The universal set ξ represents the complete set of outcomes and is called the *possibility space*.

A or *B* means event *A* occur or event *B* occurs or both occur.

$A \cup B$ means *A* or *B* and **$A \cap B$ means *A* and *B***.

Using set notation, the addition law for mutually exclusive events is $\mathrm{P}(A \cup B) = P(A) + P(B)$.

A and B are mutually exclusive when $\mathrm{P}(A \cap B)$=0 that is, when $\mathrm{A} \cap B = \emptyset$ ($\emptyset$ $-$empty set)

For non-mutually exclusive events, $\mathrm{P}(A \cup B)$ can be found by enumerating (counting) the favourable equally likely outcomes, taking care not to count any of them twice.

Example 3:

One digit is randomly selected from $1, 2, 3, 4, 5, 6, 7, 8$ and 9. Three possible events are:

A: a multiple of 3 is selected.

B: a factor of 8 is selected.

C: a prime number is selected.

a. Show that the only pair of mutually exclusive events from A, B and C is A and B, and find $\mathrm{P}(A \cup B)$.

b. Find:

- **i.** $\mathrm{P}(A \cup C)$
- **ii.** $\mathrm{P}(B \cup C)$.

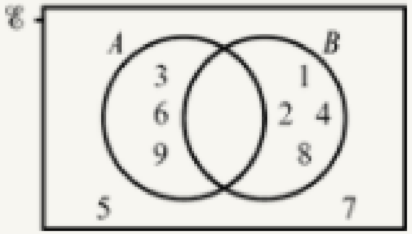
$\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

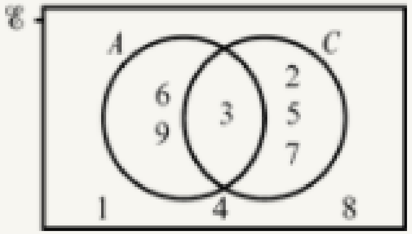
$A = \{3, 6, 9\}$, so $\mathrm{P}(A) = \frac{3}{9}$.

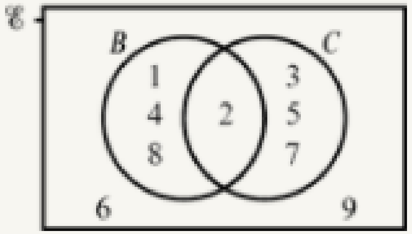
$B = \{1, 2, 4, 8\}$, so $\mathrm{P}(B) = \frac{4}{9}$.

$C = \{2, 3, 5, 7\}$, so $\mathrm{P}(C) = \frac{4}{9}$.

By listing and counting the favourable outcomes, we can find the probability for each event.







Outcomes favourable to pairs of events are shown in the three Venn diagrams opposite.

a. $A \cap B = \emptyset$, so A and B are mutually exclusive. $A \cap C \neq \emptyset$, so A and C are not mutually exclusive. $B \cap C \neq \emptyset$, so B and C are not mutually exclusive.	Two events are mutually exclusive when they have no common favourable outcomes; that is, when their intersection is an empty set.

$\begin{aligned} \mathrm{P}(A \cup B) &= \mathrm{P}(A \text{ or } B) \\ &= \mathrm{P}(A) + \mathrm{P}(B) \\ &= \frac{3}{9} + \frac{4}{9} \\ &= \frac{7}{9} \end{aligned}$	We can use the addition law because A and B are mutually exclusive events.

b. Both parts of this question can be answered using the lists of elements or the previous Venn diagrams.

i. $n(A \cup C) = n(A) + n(C) - n(A \cap C)$
$= 3 + 4 - 1 = 6$

$P(A \cup C) = P(A) + P(C) - P(A \cap C)$
$= \frac{3}{9} + \frac{4}{9} - \frac{1}{9} = \frac{6}{9}$ or $\frac{2}{3}$

Set A contains 3 of the 9 elements.

Set C contains 4 of the 9 elements.

Set A and set C have 1 of the 9 elements in common.

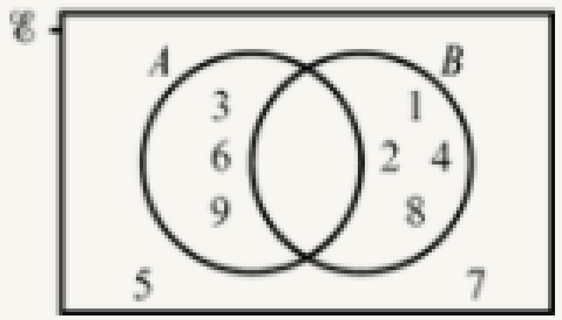
ii. $n(B \cup C) = n(B) + n(C) - n(B \cap C)$
$= 4 + 4 - 1 = 7$

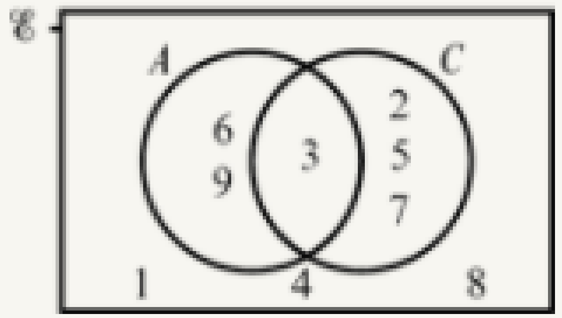
$P(B \cup C) = P(B) + P(C) - P(B \cap C)$
$= \frac{4}{9} + \frac{4}{9} - \frac{1}{9} = \frac{7}{9}$

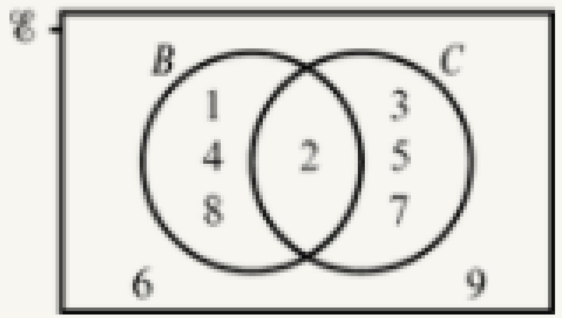
Set B contains 4 of the 9 elements.

Set C contains 4 of the 9 elements.

Set B and set C have 1 of the 9 elements in common.





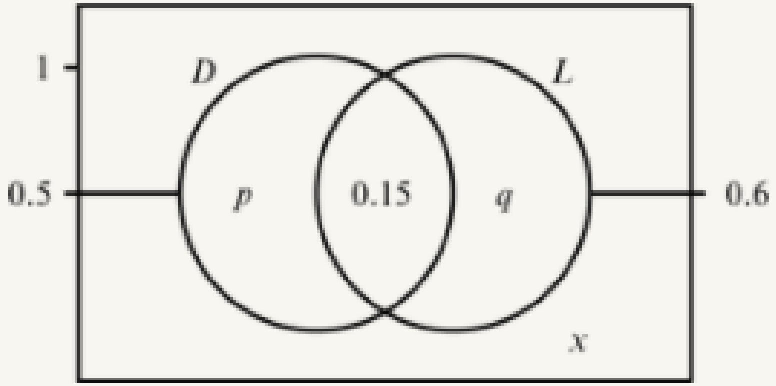


Example 4:

In a survey, 50% of the participants own a desktop (D), 60% own a laptop (L) and 15% own both.

What percentage of the participants owns neither a desktop nor a laptop?

Answer



The Venn diagram shows the given information, where p, q and x represent, respectively, the percentage that own a desktop only, a laptop only and neither of these.

$$p = 0.5 - 0.15 = 0.35$$
$$q = 0.6 - 0.15 = 0.45$$
$$x = 1 - (0.35 + 0.15 + 0.45) = 0.05 \text{ or } 5\%$$

$\therefore$ 5% of the participants own neither a desktop nor a laptop.