CHAPTER 4: PROBABILITY

AS Level

Example 1:

Esme has a bag with 5 green counters and 4 red counters.
She takes three counters at random from the bag without replacement.

Work out the probability that the three counters are all the same colour.

$\frac{1}{6}$ oe	**4**	**M3** for $\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}+\frac{4}{9}\times\frac{3}{8}\times\frac{2}{7}$ or **M2** for $\frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}$ or $\frac{4}{9}\times\frac{3}{8}\times\frac{2}{7}$ or **M1** for $\frac{5}{9},\frac{4}{8},\frac{3}{7}$ seen or $\frac{4}{9},\frac{3}{8},\frac{2}{7}$ seen If 0 scored, **SC1** for $\frac{5^3+4^3}{729}$ oe

Example 2:

A box contains 20 packets of potato chips.

6 packets contain barbecue flavoured chips.
10 packets contain salt flavoured chips.
4 packets contain chicken flavoured chips.

(a) Maria takes two packets at random **without replacement**.

(i) Show that the probability that she takes two packets of salt flavoured chips is $\frac{9}{38}$.

$\frac{10}{20} \times \frac{9}{19}$ oe	**M2**	**B1** for $\frac{9}{19}$ oe seen

(ii) Find the probability that she takes two packets of different flavoured chips.

$\frac{62}{95}$ oe	**4**	**M3** for $\frac{6}{20} \times \frac{14}{19} + \frac{10}{20} \times \frac{10}{19} + \frac{4}{20} \times \frac{16}{19}$ oe or $1 - \frac{6}{20} \times \frac{5}{19} - \frac{10}{20} \times \frac{9}{19} - \frac{4}{20} \times \frac{3}{19}$ oe or **M2** for the sum of two products of different flavours isw or **M1** for one correct product of different flavours isw

A box contains 20 packets of potato chips.

6 packets contain barbecue flavoured chips.
10 packets contain salt flavoured chips.
4 packets contain chicken flavoured chips.

(b) Maria takes three packets at random, **without replacement**, from the 20 packets.

Find the probability that she takes **at least two** packets of chicken flavoured chips.

$\frac{5}{57}$ oe	**3**	**M2** for $N \times \left(\frac{4}{20} \times \frac{3}{19} \times \frac{16}{18}\right) + \frac{4}{20} \times \frac{3}{19} \times \frac{2}{18}$ oe or for $3\left(\frac{4}{20} \times \frac{3}{19} \times \frac{16}{18}\right)$ oe or $1 - \{N \times \left(\frac{4}{20} \times \frac{16}{19} \times \frac{15}{18}\right) + \frac{16}{20} \times \frac{15}{19} \times \frac{14}{18}\}$ oe or **M1** for $\frac{4}{20} \times \frac{3}{19} \times \frac{k}{18}$ oe seen

4.1 Experiments, events and outcomes

Experiments, events and outcomes

The result of an experiment is called an **outcome** or **elementary event**, and a combination of these is known simply as an event.

Rolling an ordinary fair die is an experiment that has six possible outcomes: 1, 2, 3, 4, 5 or 6.

Obtaining an odd number with the die is an event that has three **favourable** outcomes: 1, 3 or 5.

5000₮

Sum of dies	Money
4	500
5	600
6	700
7	800
8	900
…	…
24	50000

Random selection and equiprobable events

The purpose of selecting object at **random** is to ensure that each has the same chance of being selected. This method of selection is called **fair** or **unbiased**, and the selection of any particular object is said to be **equally likely** or **equiprobable**.

Bais/Unfair

Bais/Unfair

Bais/Unfair

When one object is randomly selected from n objects,

$$\boldsymbol{P(selecting\ any\ particular\ object) = \frac{1}{n}}$$

$$P(2) = \frac{1}{6}$$

Random selection and equiprobable events

The probability that an event occurs is equal to the proportion of equally likely outcomes that are favourable to the event.

$$P(event) = \frac{Number\ of\ favourable\ equally\ likely\ outcomes}{Total\ number\ of\ equally\ likely\ outcomes}$$

Example 3: Consider randomly selecting 1 student from a group of 19, where 11 are boys and 8 are girls.

Event/Outcome	Probability	Description
Selecting any particular boy	$\frac{1}{19}$	These three outcome are equally likely.
Selecting any particular girl	$\frac{1}{19}$	
Selecting any particular student	$\frac{1}{19}$	
Selecting a boy	$\frac{11}{19}$	11 of the 19 equally likely outcomes are favorable to this event.
Selecting a girl	$\frac{8}{19}$	8 of the 19 equally likely outcomes are favourable to this event.

The word **particular** species one object. It does not matter whether that object is a boy, a girl or a student.

Exhaustive events

A set of events that contains all the possible outcomes of an experiment is said to be exhaustive. In the special case of event A and its **complement**, not A, the sum of their probabilities is 1 because one them is certain to occur. Recall that the notation used for complement of set A is $\boldsymbol{A'}$.

$$\boldsymbol{P(A) + P(not\ A) = 1}$$

$$\boldsymbol{or}$$

$$\boldsymbol{P(A) + P(A') = 1}$$

Example 4:

Experiment	Exhaustive events		Probabilities
	A	A′	
Toss a fair coin	Heads	Tails	$\frac{1}{2}+\frac{1}{2}=1$
Roll a fair die	Less than 2	2 or more	$\frac{1}{6}+\frac{5}{6}=1$
Play a game of chess	Win	Not win	$\frac{1}{2}+\frac{1}{2}=1$

Trails and expectation

Each repeat of an experiment is called **a trial**. The proportion of trails in which an event occurs is its **relative frequency**, and we can use this an estimate of the probability that the event occurs.

If we know the probability of an event occurring, we can estimate the number of times it is likely to occur in a series of trials. This is a statement of our **expectation.**

In n trials, event A is expected to occur $n \times P(A)$ times.

Example 5:

The probability of rain on any particular day in mountain village is 0.2. On how many days is rain not excepted in a year of 365 days?

$$P(does\ not\ rain) = 1 - 0.2 = 0.8$$

$$365 \times 0.8 = 292$$