Probability & Statistics

Chapter 2

Measures of central tendency

Cambridge AS level

THREE TYPES OF AVERAGE

There are three measures of central tendency that are commonly used to describe the **average** value of a set of data. These are the **mode**, the **mean** and the **median**.

- The mode is the most commonly occurring value.
- The mean is calculated by dividing the sum of the values by the number of values.
- The median is the value in the middle of an ordered set of data.

We use an average to **summarise** the values in a set of data. As a representative value, it should be fairly central to, and typical of, the values that it represents.

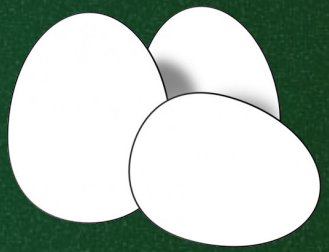
THREE TYPES OF AVERAGE

If we investigate the annual incomes of all the people in a region, then a single value (i.e. an average income) would be a convenient number to represent our findings. However, choosing which average to use is something that needs to be thought about, as one measure may be more appropriate to use than the others.

THREE TYPES OF AVERAGE

Deciding which measure to use depends on many factors.

Although the mean is the most familiar average, a shoemaker would prefer to know which shoe size is the most popular (i.e. the mode).



A farmer may find the median number of eggs laid by their chickens to be the most useful because they could use it to identify which chickens are profitable and which are not.

As for the average income in our chosen region, we must also consider whether to calculate an average for the workers and managers together or separately; and, if separately, then we need to decide who fits into which category.

2.1 THE MODE AND THE MODAL CLASS

As you will recall, a set of data may have more than one mode or no mode at all.

The following table shows the scores on 25 rolls of a die, where 2 is the mode because it has the highest frequency.

Score on die	1	2	3	4	5	6
Frequency (f)	5	6	5	3	2	4

2.1 THE MODE AND THE MODAL CLASS

In a set of grouped data in which raw values cannot be seen, we can find the **modal class**, which is the class with the highest frequency density.

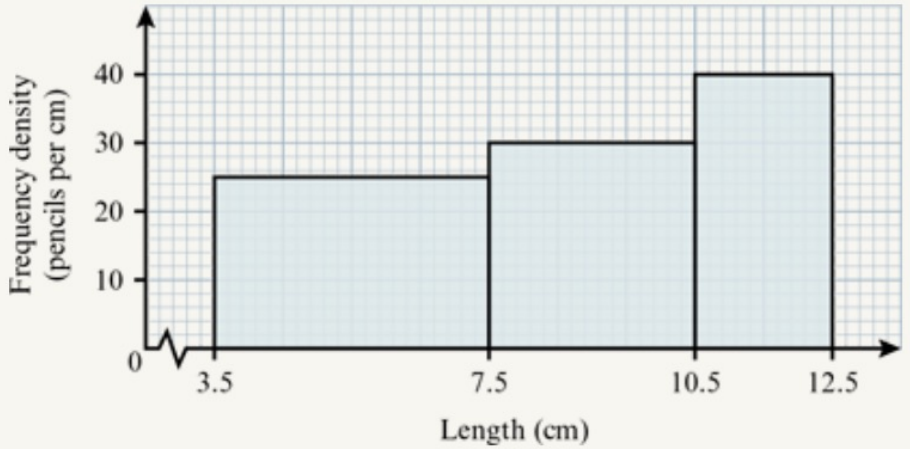
KEY POINT: In histograms, the modal class has the greatest column height. If there is no modal class then all classes have the same frequency density

Example 1:

Find the modal class of the 270 pencil lengths, given to the nearest centimetre in the following table.

Length (x cm)	No. pencils (f)
4–7	100
8–10	90
11–12	80

Length (x cm)	No. pencils (f)	Width (cm)	Frequency density
$3.5 \leqslant x < 7.5$	100	4	$100 \div 4 = 25$
$7.5 \leqslant x < 10.5$	90	3	$90 \div 3 = 30$
$10.5 \leqslant x < 12.5$	80	2	$80 \div 2 = 40$



The modal class is 11-12cm (or, more accurately, $10.5 \leqslant x < 12.5$).

Example 2:

Two classes of data have interval widths in the ratio 3:2 . Given that there is no modal class and that the frequency of the first class is 48, find the frequency of the second class.

$48 : x = 3 : 2$

$3x = 96$

$x = 32$

TIP: In the special case where all classes have equal widths, frequency densities are proportional to frequencies, so the modal class is the class with the highest frequency.

Homework

Page 29 – Exercise 2A

1 Find the mode(s) of the following sets of numbers.

a 12, 15, 11, 7, 4, 10, 32, 14, 6, 13, 19, 3

b 19, 21, 23, 16, 35, 8, 21, 16, 13, 17, 12, 19, 14, 9

2 Which of the eleven words in this sentence is the mode?

3 Identify the mode of x and of y in the following tables.

x	4	5	6	7	8
f	1	5	5	6	4

y	−4	−3	−2	−1	0
f	27	28	29	27	25

4 Find the modal class for x and for y in the following tables.

x	0–	4–	14–20
f	5	9	8

y	3–6	7–11	12–20
f	66	80	134

5 A small company sells glass, which it cuts to size to fit into window frames. How could the company benefit from knowing the modal size of glass its customers purchase?

6 Four classes of continuous data are recorded as 1–7, 8–16, 17–20 and 21–25. The class 1–7 has a frequency of 84 and there is no modal class. Find the total frequency of the other three classes.

7 Data about the times, in seconds, taken to run 100 metres by n adults are given in the following table.

Time (x s)	$13.6 \leqslant x < 15.4$	$15.4 \leqslant x < 17.4$	$17.4 \leqslant x < 19.8$
No. adults (f)	a	b	27

By first investigating the possible values of a and of b, find the largest possible value of n, given that the modal class contains the slowest runners.

8 Three classes of continuous data are given as 0–4, 4–10 and 10–18. The frequency densities of the classes 0–4 and 10–18 are in the ratio 4 : 3 and the total frequency of these two classes is 120. Find the least possible frequency of the modal class, given that the modal class is 4–10.